Scalable and Accurate Variance Estimation (SAVE) for Joint Bayesian Compressed Sensing

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TARGET AUDIENCE: Image reconstruction and compressed sensing; neuroimaging scientists and clinicians.

PURPOSE: The far reaching adoption of compressed sensing (CS) for clinic MRI hinges on the ability to accurately produce images in a reasonable time-frame. Multiple contrast studies have been successfully combined with joint Bayesian image reconstruction [1,2,3] in order to exploit mutual information for improved image quality. However, these techniques have prohibitive computational requirements. We leverage hierarchical matrix analysis and compression schemes [4,5,6] to facilitate scalable and accurate CS reconstruction. Our approach is over 100x faster than other multiple contrast approaches [1,2] while still improving image accuracy by over 35% compared to single image CS techniques [7].

METHOD: The proposed method captures critical information from point spread functions to be used within our variance estimation framework (SAVE) for Bayesian reconstruction.

Joint Bayesian CS: Based upon the Bayesian CS formulation described in [1,2,3] we consider L under-sampled images $\{x_i\}_{i=1,L}$, with associated image and k-space gradients $\{\delta_i\}_{i=1,L}$ and $\{z_i\}_{i=1,L}$. The data are modeled to be corrupted by complex Gaussian noise with variance σ^2, assuming an under-sampled Fourier operator F_Ω. The L images are coupled using a Gaussian prior and through Bayes' rule, the posterior becomes $p(\delta_i|z_i,\gamma) = N(\mu_i, \sum)$. Here, we need to iteratively solve for the diagonal entries of (i) $\sum = \left(\Gamma^{-1} + \sigma^{-2}F_\Omega^H F_\Omega\right)^{-1}$ and the linear solution of (ii) $\mu_i = \Gamma F_\Omega^H\left(\sigma^2 I + F_\Omega \Gamma F_\Omega^H\right)^{-1} z_i$, where the hyperparameters $\Gamma = diag(\gamma)$ are updated during each iteration using $\gamma_t \leftarrow \|\mu_t\|^2/(L - L\sum_{tt}/\gamma_t)$.

Scalable and Accurate Variance Estimation: Attempts have been made to solve Eq. (i) through low rank approximation [2] using a Lanczos based algorithm [8]. These methods have been shown to significantly reduce normalized root-mean-square error (NRMSE) when compared to the total variation (TV) penalty CS algorithm by Lustig et al. [7]. However, the method suffers from prohibitive computational time. We consider an alternative approach that approximates point spread functions to exploit sparse matrix methods. Fig. 1 shows a R=4 random under-sampling pattern with associated point spread function. The significant entries can be used to approximate the operator $\left(F_\Omega^H F_\Omega\right)$ by a sparse matrix. The correlation inverse can then be hierarchically compressed [6]. To solve Eq. (i) efficiently, the compressed matrix is used within a fast variance estimation technique similar to [4,5]. Eq. (ii) can be solved using standard GMRES.

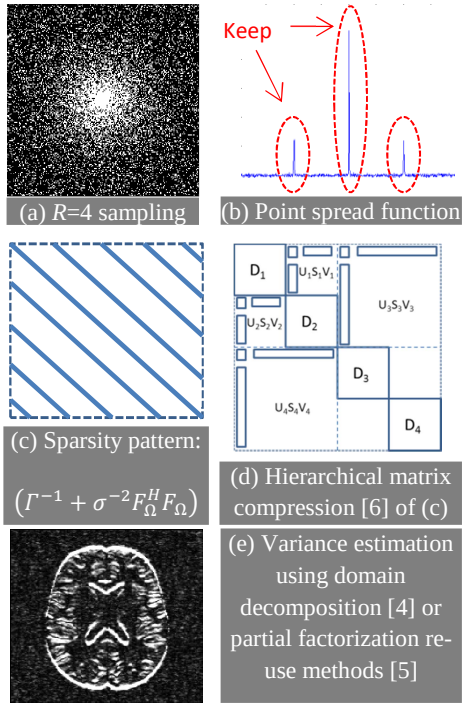
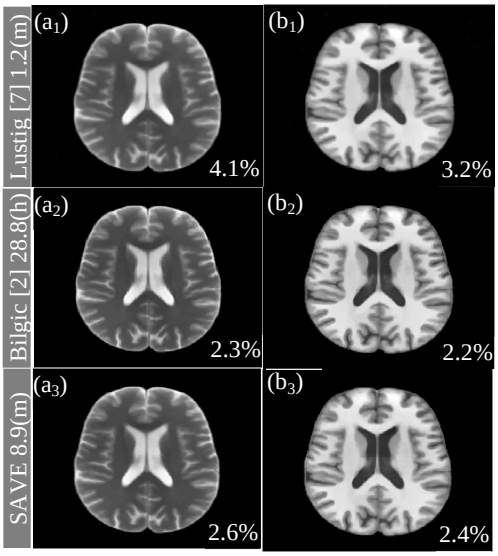


Fig. 1. (a,b) point spread function and sparse approximation (c) sparsity pattern for inverse correlation (d) compressed matrix form (e) scalable variance estimation

RESULTS: We consider the SRI24 atlas [9] that features 200×200 proton density (PD), T2, and T1 weighted scans. We under-sample all with R = 4 (Fig. 1) and a zero fill-in image is supplied as prior. Fig. 2 shows the accuracy and run-time for Lustig et al. TV [7], along with 5 iterations for both Bilgic et al. [2] and the proposed SAVE method using MATLAB on a AMD Opteron 6282SE.

DISCUSSION: Figs. 2(a,b) show the T2 and T1 weighted reconstructions respectively with the associated NRMSE. For the T2, T1, and PD images, Lustig et al. [7] required 1.2min with mean error of 3.26%. Bilgic et al. [2] produced the lowest error 1.94% but with prohibitive run-time of 28.8hours. The SAVE method achieved 2.15% mean error in under 9min. The computational performance of SAVE is summarized in Fig 2(c).



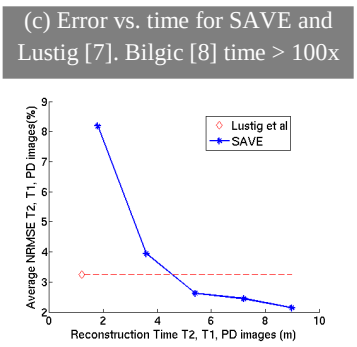


Fig. 2. (a,b) accuracy (NRMSE) for T2 and T1 images. Time includes PD image (not shown), [7] and SAVE in min and for [2] in hours (c) accuracy-time trade-off for [7] and SAVE.

CONCLUSION: We present a variance estimation framework for joint Bayesian CS reconstruction of multiple contrast images. Our approach offers over 100x speed-up compared to [1,2] while preserving image accuracy. Our scalable and accurate framework for CS reconstruction is applicable to a wide range of under-sampling strategies and acquisitions.

REFERENCES: [1] Bilgic et al., MRM 2011; [2] Bilgic et al., ISMRM 2012; [3] Wipf and Rao, IEEE T Signal Proces 2007; [4] Cauley et al., JAP 2007; [5] Li et al., JCP 2008; [6] Xia et al., SIMAX 2009; [7] Lustig et al. MRM 2007; [8] Seeger et al. MRM 2010; [9] Rohlfing et al., Hum Brain Mapp 2010;