

Improved *l1*-SPIRiT Using Tensor-Based Sparsity Basis

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Introduction: The combination of parallel imaging (PI) and compressed sensing (CS) has been widely studied in the MRI field. One typical method is to implement CS as a separate stage either before or after the PI step. Two major issues exist in this method. Firstly, noise and errors introduced in one operation can be augmented in the other procedure. Secondly, CS is performed individually on the data from each coil, i.e. 2D sparsity bases are used coil by coil, thus only the intra-coil redundancy is exploited while the inter-coil redundancy is omitted and wasted. In this work, we propose a novel CS-PI framework to study these two issues. To avoid the noise and error augmentation, a filter criterion is set that the true collected measurements are always kept while the estimated measurements are updated in a circulatory feeding process. To exploit both the intra- and inter-coil redundancies, signals from all the array coils are integrated to form a three-dimensional tensor, thus a novel tensor-based sparsity basis can be developed for the CS reconstruction. The proposed CS-PI algorithm has been successfully applied to a brain imaging study.

Theory: The high order singular value decomposition (HOSVD)¹ is a tensor decomposition method. Given a third-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$, it can be decomposed as $\mathcal{X} = \mathcal{S} \times_1 U_1 \times_2 U_2 \times_3 U_3$, where $\mathcal{S} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ is named as the core tensor and $U_k \in \mathbb{C}^{I_k \times I_k}$, $k = 1, 2, 3$, is the supporting matrix. The core tensor \mathcal{S} is *all-orthogonal* and *ordering*, and these two attributions make \mathcal{S} have as many elements zero or as small as possible. In CS, this means that the core tensor \mathcal{S} is *sparse*. In this way, we can treat HOSVD as a sparsifying transform, which

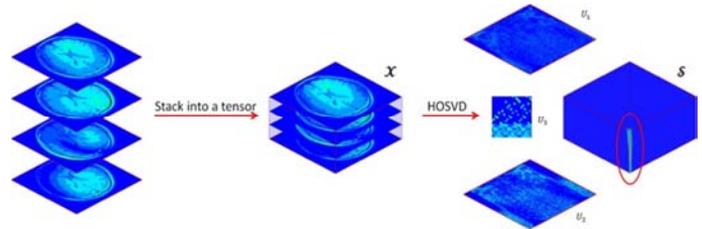


Fig. 1 HOSVD sparsifying transform for a four-coil brain dataset. Left: images collected in four coils are stacked into a tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$. Right: the tensor is decomposed into a core sparse tensor $\mathcal{S} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ and three supporting matrices $U_k \in \mathbb{C}^{I_k \times I_k}$, $k = 1, 2, 3$. The nonzero values in tensor \mathcal{S} are limited in the small corner of the tensor (see red circle).

transforms a dense tensor \mathcal{X} into a sparse tensor \mathcal{S} with supporting matrices U_1 , U_2 and U_3 . In the CS step, the 2D signals from each coil are stacked into a third-order tensor, and then the image space of each coil and the correlation cross all the coils are simultaneously sparsified by HOSVD. Compared with conventional 2D wavelet transforms² on each slice, e.g., this new sparsifying operation offers a more natural and compact way to sparsify the image data. A visualization of HOSVD sparsifying transform for a four-coil dataset is shown in Fig. 1.

Based on the above HOSVD operation for all the coil data, the proposed CS-PI scheme is a three-step reconstruction process (see Fig. 2). In step 1, given the under-sampled Fourier signals collected from multiple coils, a nonlinear conjugate gradient descent algorithm (NLCG) is first performed for CS reconstruction. In step 2, using the CS result from step one, the *l1*-SPIRiT algorithm is carried out for parallel imaging reconstruction with the projection over convex sets (POCS) approach. In step 3, the image data will be combined with *l1*-SPIRiT estimated result and measured data. In steps 2 and 3, the intermediate results are filtered in the way that only the non-sampled elements by coils are kept and then combined with the signals collected by array coils. In this way, noise and errors from the CS/PI stage can be partly eliminated in the subsequent PI/CS stage.

Method: We performed CS using the SparseMRI² software package and PI reconstruction using the *l1*-SPIRiT³ software package. To verify the effectiveness of the proposed noise and error filter criterion, the reconstructed results of pure *l1*-SPIRiT were also recorded. To illustrate the superiority of the HOSVD-based sparsity basis, the two-dimensional wavelet sparsity basis was also tested, which can only exploit intra-coil redundancies. In experiments an eight-coil data was used⁴, which was acquired using a 2D spin echo sequence with echo time = 11 ms, pulse repetition time = 700 ms, matrix size = 256×256 and field of view = 220 mm². The Poisson disc sampling patterns were used to collect partial signals in Fourier domain.

Results: Under reduction factor of 3, the MRI image was reconstructed using the *l1*-SPIRiT method with an NMSE of 0.267, while using the proposed method with two-dimensional wavelet sparsity basis and HOSVD-based sparsity basis, the reconstruction errors (NMSE) are 0.127, and 0.121, respectively. Under reduction factor of 5, the NMSEs of the three methods were recorded as 0.567, 0.239 and 0.183 respectively. Figure 3 shows the reconstructed images with a reduction factor of 5 and the corresponding residual error maps (5× amplification).

Discussion and Conclusion: In this work, a novel CS-PI scheme has been developed. In this new method, a noise and error filter criterion was used to improve the sequential combination method of PI and CS. More importantly, a third-order singular value decomposition was used as a novel sparsity basis to fully exploit and intra- and inter-coil redundancies in signal sparsity. The case study clearly indicates that the new method has superior performance in terms reconstruction quality for static imaging. With forth-order singular value decomposition, the proposed scheme can also be used for *k-t* CS-PI dynamic imaging studies.

References: 1. Lathauwer LD, Moor BD, Vandewalle J. SIAM J Matrix Anal A. 2000; 21 (4): 1253–78. 2. Lustig M, Donoho DL, Pauly JM.. MRM. 2007; 58 (6): 1182–95. 3. Lustig M, Pauly JM. MRM. 2010; 64 (2): 457–71. 4. Chang Y, Liang D, Ying L. MRM. 2012; 68:730–40.

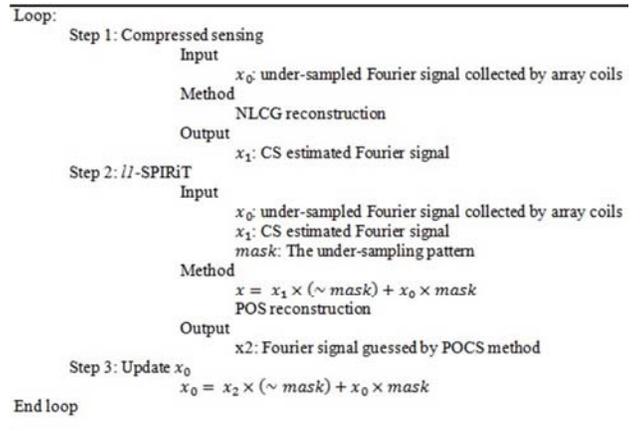


Fig. 2 Three-step iterative process for the sequential CS-PI reconstruction.

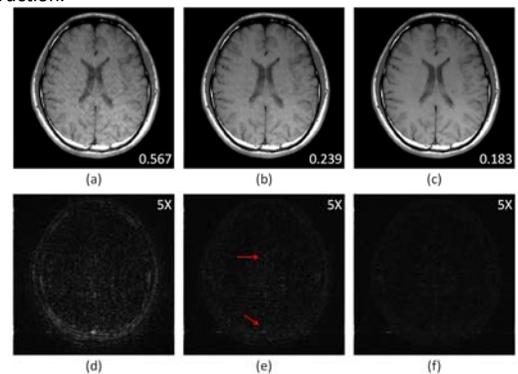


Fig. 3 Brain images reconstructed with a reduction factor of 5 and the corresponding residual error maps (5× amplification). The first column: *l1*-SPIRiT. The second column: the proposed method with wavelet sparsity basis. The Third column: the proposed method with HOSVD sparsity basis. The NMSE values are labelled in each recovered image.