Magnetic Susceptibility and Field Map Estimation in fMRI time series using a High Resolution Static Field Map

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Introduction: Functional MRI (fMRI) time series data are mostly acquired using echo planar imaging (EPI) sequence which provides the advantage of high temporal resolution. Yet, EPI is highly sensitive to the magnetic field inhomogeneity and results in geometric distortions and non-uniform blur in the acquired images [1]. A static field-inhomogeneity map may be measured before or after an fMRI session to correct for such distortions, but it does not account for magnetic field changes due to head motion during the time series acquisition. In practice, the field map dynamically changes with head motion during the scan, and such a changing field leads to variations in geometric distortions. A previous retrospective approach of approximating a dynamic field map by applying rigid body transformations to an observed static field map may not be sufficient in the presence of significant out-of-plane rotations since the field inhomogeneity may change nonlinearly [2]. We model in this work the field inhomogeneity with two specific, the object and the scanner dependent terms. We assume that the scanner-specific field remains unchanged and independent of the head motion. The object-specific term varies with the object's magnetic susceptibility and orientation, i.e., head position with respect to B0. Thus, the simple transformation of the acquired field map may not yield an accurate field map. Our approach in this study is to retrospectively estimate the object's magnetic susceptibility (χ) map from an observed high-resolution static field map using an estimator derived from a probability density function of non-uniform noise. This approach is capable of finding the susceptibility map regardless of the wrapping effect. To compute the dynamic field maps, we apply rigid body motion to the χ-map estimate, and apply 3D susceptibility voxel convolution (SVC) which is a physics based discrete convolution model for computing χ induced field-inhomogeneity given a 3D χ-map [2].

Method: A field map ΔB is obtained from a pair of (complex-valued) images acquired at different echo times, T_{E1} and T_{E2} (a dual-echo sequence was used [3,4]). The field inhomogeneity (ΔB) is measured from the phase difference θ between the two images as $\theta[\mathrm{rad}] = \angle(\bar{I}_{\mathrm{TE1}} \cdot I_{\mathrm{TE2}}) = \gamma \Delta B \Delta T_{\mathrm{E}} - 2\pi n$, where γ is the gyromagnetic ratio, $-2\pi n$ represents the wrapping effect with some integer n so that θ stays in $[-\pi, \pi]$, and $\Delta T_{\mathrm{E}} = T_{\mathrm{E2}} - T_{\mathrm{E1}}$. We denote that $\Delta B = \Delta B_\chi + \Delta B_s$, where ΔB_χ and ΔB_{s} are the object-specific susceptibility (χ)-induced field and the scanner specific terms, respectively. The χ-induced field produces $\Delta B_\chi = h * \chi + \Delta B_{\mathrm{e}}$, where h is the 3D SVC kernel [3,5] and ΔB_{e} is the error due to χ induced secondary field effect. We have a data model as

$$\theta(\mathbf{r}) = \gamma\{\Delta B_{\mathrm{s}}(\mathbf{r}) + \Delta B_\chi(\mathbf{r}) + \Delta B_{\mathrm{e}}(\mathbf{r})\}\Delta T_{\mathrm{E}} - 2\pi n(\mathbf{r}) + \varepsilon(\mathbf{r}) = \theta_{\mathrm{s}}(\mathbf{r}) + \gamma \Delta T_{\mathrm{E}} \cdot h(\mathbf{r}) * \chi(\mathbf{r}) - 2\pi n(\mathbf{r}) + \varepsilon(\mathbf{r}) \quad \in [-\pi, \pi] \tag{1}$$

where $\mathbf{r} = [x, y, z, q]$ represents the spatial voxel coordinate and the coil index (q) of the multi-coil scanner, ε is zero-mean (real-valued) additive noise, and θ_ϵ $(= \gamma(\Delta B_{\mathrm{s}} + \Delta B_{\mathrm{e}})\Delta T_{\mathrm{E}})$. Assuming that $\Delta B_\chi > \Delta B_{\mathrm{s}} > \Delta B_{\mathrm{e}}$ and smooth across the space, often, θ_ϵ is separated from θ by applying a low-pass filter, e.g. [6,7], then the susceptibility map χ is estimated by deconvolution using the SVC kernel h where the noise ε is assumed to be Gaussian. By this assumption the noise ε may become non-uniform and necessitates the estimation of local noise variance and phase unwrapping, which is not an easy task due to the non-uniform noise and the complex structure of the human anatomy. These issues can be handled by introducing appropriate noise statistics. Assuming that the noise ridden on the two images, I_{TE1} and I_{TE2}, is *i.i.d.* zero-mean complex Gaussian noise with variance of σ^2, we derive the provability density function (PDF) of the noise ε in (1) as

$$p(\varepsilon(\mathbf{r})) \sim \exp\{\kappa(\mathbf{r}) \cdot \cos \varepsilon(\mathbf{r})\} \qquad \text{with} \quad \kappa(\mathbf{r}) = \frac{|I_{\mathrm{TE1}}(\mathbf{r})|^2 |I_{\mathrm{TE2}}(\mathbf{r})|^2}{(|I_{\mathrm{TE1}}(\mathbf{r})|^2 + |I_{\mathrm{TE2}}(\mathbf{r})|^2)\sigma^2 + \sigma^4}. \tag{2}$$

It is advantageous that the term $2\pi n$ has no effect in $\cos(\cdot)$ and the term κ eliminates noisy data (i.e. phase samples in the region where the coil sensitivity is low) by giving small weights. Having introduced the data model (1) and the noise PDF (2), we have the following maximum likelihood estimator for χ and θ_ϵ with regularizations as

$$\max_{\chi, \theta_s} \sum_{\text{for all } \mathbf{r}} \kappa(\mathbf{r}) \cdot \cos\big(\theta(\mathbf{r}) - \theta_\epsilon(\mathbf{r}) - \gamma \Delta T_E \cdot h(\mathbf{r}) * \chi(\mathbf{r})\big) + \sum_{i = \{x,y,z\}} \big\{\mu_1 \cos\big(\Upsilon_i(\mathbf{r}) * \theta_\epsilon(\mathbf{r})\big) - \mu_2 |\Gamma_i(\mathbf{r}) * \chi(\mathbf{r})|\big\} \tag{3}$$

where μ_1 and μ_2 are the regularization parameters for θ_ϵ and χ, respectively, andΓ_i and Υ_i are the filters of the first and second derivative, respectively, along i-axis for $i = \{x, y, z\}$. We chose the filters based on the assumptions that the scanner-specific field is piecewise smooth and the χ-map is piecewise constant across the space. We estimate θ_{s} and χ by the steepest descent method with updating θ_ϵ and χ iteratively. We initialize χ by first making a binary image of the body tissue and air from I_{TE1} of one coil channel then filling the literature χ values of water and air in. We also initialize θ_ϵ with the residuals, i.e. $\hat{\theta}_\epsilon^{(0)} = \theta - \gamma \Delta T_{\mathrm{E}} \cdot h * \hat{\chi}^{(0)}$ with $\hat{\chi}^{(0)}$ the initial χ-map. The 2nd order term ΔB_{e} can be estimated by treating it as an error in the field map computation on the distortion correction and iteratively refines the computed field map as described in our previous work [8].

Experiments: We estimated χ maps from two data sets from a homogeneous phantom and a real human subject. Both data were taken by a dual echo sequence with the voxel resolution $1 \times 1 \times 1[\mathrm{mm}^3]$ and $TE1/TE2 = 4.0/5.5$. In Fig.1 we show (a) magnitude images and (b) measured field maps from one channel out of 13 channel data, (c) estimated susceptibility maps ($\hat{\chi}$) and (d) field error $\hat{\theta}_\epsilon$, (e) residuals ($= \theta - \hat{\theta}_\epsilon - \gamma \Delta T_E \cdot h * \hat{\chi}$). For the homogeneous phantom, the known χ value was approximately equal to the water's ($\approx 9.06 \times 10^{-6}$), and the estimated χ values was 8.95×10^{-6}. Using the estimated χ-map, (f) χ-induced field map was computed by SVC.

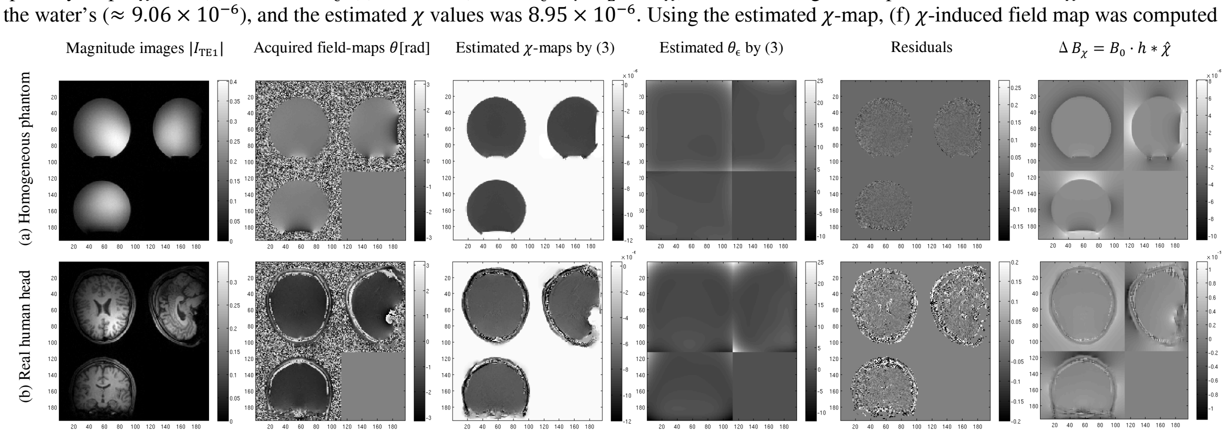

Figure 1. Susceptibility map estimation using a homogeneous phantom (upper column) and a human MRI data (lower column):(a) magnitude images, (b) acquired field maps of one coil channel, (c) estimated $\boldsymbol{\chi}$ maps ($\hat{\boldsymbol{\chi}}$), (d) error $\hat{\boldsymbol{\theta}}_\epsilon$, and (e) € the residuals ($= \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_\epsilon - \boldsymbol{\gamma \Delta T_E \cdot h} * \hat{\boldsymbol{\chi}}$) and (f) the computed $\boldsymbol{\chi}$-induced field

Conclusion and Future works: We derived an effective PDF of the noise ridden on the phase image and a susceptibility-map estimator based on the PDF. Our estimator is advantageous that no segmentation of human-tissue/air or phase unwrapping is necessary and the noisy samples are removed by the weight function κ in the PDF. The effectiveness of our method has been demonstrated in the experimental result. Further study will include the computation of χ-induces field map affected by the head motion for the reconstruction of the geometrically distorted images in fMRI.

References

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