Investigating anisotropic magnetic susceptibility effects in model systems

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Introduction Anisotropic magnetic susceptibility effects are of increasing interest in the investigation of contrast in nerve tissue because of the anisotropic magnetic properties of myelin. In particular, anisotropic susceptibility has been proposed as the underlying cause of the orientation dependence of R_2* relaxation rates and frequency offsets in the human brain at high field [1]. The signature of anisotropic susceptibility has been demonstrated in phase measurements on post mortem tissue [2] and forms the basis of susceptibility tensor imaging [3]. Although correct mathematical expressions for calculating the field perturbation due to structures composed of anisotropic susceptibility have been described [3,4] the effects of such structures have often been modelled using a simplified approach in which the anisotropy is represented by allocating the material an isotropic susceptibility whose magnitude depends on its orientation to the field [1,4]. Here we demonstrate using theory and experiment that this approximation leads to errors in predicting the frequency perturbation due to anisotropic structures. We also show that the correct calculation predicts interesting behaviour in a hollow cylinder model of myelinated nerve fibers, and go on to demonstrate this behaviour experimentally in a simple model system.

Theory We consider material with anisotropic magnetic susceptibility that is described by a cylindrically symmetric tensor as shown in Eq. 1, where χ_I and χ_A (<< 1) represent the isotropic and anisotropic components of the susceptibility. If this material is exposed to a magnetic field $\mathbf{H} = H_0\mathbf{k}$ with the principal axis of the susceptibility tensor lying in the x-z plane and making an angle, Θ, to the field direction, the induced magnetization is given by $\mathbf{M} = H_0\left(0.75\chi_A \sin 2\Theta\, \mathbf{i} + \left(\chi_I + \chi_A(1 - 1.5\sin^2\Theta)\right)\mathbf{k}\right)$ $(Eq. 2)$. We can calculate the z-component of the field perturbation (in spherical polar co-ordinates) produced by this magnetization in the usual manner yielding a standard dipole term varying as $D = M_z\,(3\cos^2\theta - 1)/4\pi r^3$ due to M_z and an additional term varying as $E = 1.5\, M_x \sin 2\theta \cos\phi/4\pi r^3$ due to M_x. Modelling the effect of the anisotropy by allocating the material an isotropic susceptibility whose magnitude depends on orientation to the field produces the correct form of M_z, and associated dipole field, but neglects the effect of M_x.

$$\underline{\underline{\chi}} = \chi_I \underline{\underline{I}} + \begin{bmatrix} -\chi_A/2 & 0 & 0 \\ 0 & -\chi_A/2 & 0 \\ 0 & 0 & \chi_A \end{bmatrix} \quad Eq.[1]$$

Now considering a simple model of the myelin sheath, consisting of a long hollow cylinder (inner /outer radius = r_i/r_o) composed of material of anisotropic susceptibility such that the principal component of the tensor is radially-oriented, full calculation of the field perturbation [5] yields a uniform field inside the sheath (where $r<r_i$) of magnitude $0.75\chi_A \sin^2\Theta \ln(r_o/r_i)$ while outside the cylinder (where $r>r_o$) the field varies as $\cos 2\phi\left(r_o^2 - r_i^2\right)/2r^2$ with an amplitude given by $(\chi_I + 0.25\chi_A)\sin^2\Theta$, where Θ is the angle between the cylinder and the field. Alternatively, if we represent the anisotropy by using an orientation-dependent isotropic susceptibility, the calculation yields a field with the same form of spatial variation outside the cylinder, but with amplitude that depends upon $\left(\chi_I - 0.5\times\chi_A\left(1 - 0.5\sin^2\Theta\right)\right)\sin^2\Theta$. This approximation also gives zero field inside the cylinder. The two approaches thus make different predictions about the form of field variation, which we can test experimentally. If the wall of the cylinder is thin enough that $r_o - r_i = t$ (<< r_o, r_i), then $\ln(r_o/r_i) \sim t/r_i$ and the internal field scales as the inverse of the cylinder radius.

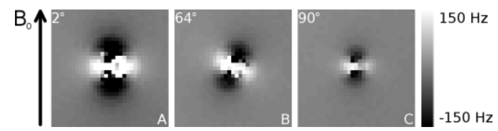

Figure 1 *Field variation produced in the x-z plane by the PGS stack at different angles to the field (2°, 64° & 90°)*

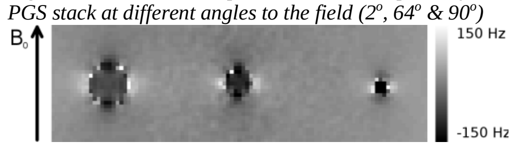

Figure 2 *Field variation produced by PGS covered cylinders (15, 10 & 5 mm dia.) when perpendicular to B_0*

Methods A dual-echo, field mapping sequence (1.5 mm resolution) was used at 3 T to measure the field perturbation due to different structures produced from 25 or 70 μm thick pyrolitic graphite sheet (PGS-Panasonic EYGS1218-03/07). PGS is strongly diamagnetic and magnetically anisotropic, with a cylindrically symmetric tensor whose principal component is normal to the sheet. ***Experiment 1***: 5-mm-diameter disks of PGS were stuck together to form ~5mm-thick stacks, which were embedded in an 18 cm diameter agar filled sphere. Field maps were formed with the PGS stacks oriented at 5 different angles to the field. In each case, the field variation in a spherical shell around the stack was analysed to identify the amplitude of the field components due to M_z (D) and M_x (E). These amplitudes were then plotted against sin 2Θ or $\sin^2\Theta$ to allow evaluation of the magnitude of χ_I and χ_A using Eq. 2. ***Experiment 2*** Glass tubes of 3 different diameters (5, 10 and 15 mm) were covered in a layer of 25 μm thick PGS producing a structure mimicking the hollow cylinder model. The tubes were filled with agar and embedded in an agar sphere. Field maps were generated with the tubes at 4 different orientations to the field. ***Experiment 3*** a single glass tube of 15 mm diameter was covered with a layer of 70 μm thick PGS and embedded in the agar-filled sphere. External field perturbations were measured with the tube at 6 different angles to the field.

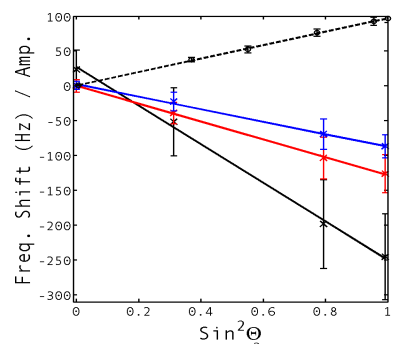

Figure 3 *Variation with $\sin^2\Theta$ of internal field for 15, 10 & 5 mm diameter tubes (blue, red & black ×; error bars = s.d.) and external field (black o; bars = fit errors).*

Results ***Experiment 1*:** Figure 1 shows portions of the field maps measured in the x-z plane through the centre of a PGS stack, with the stack oriented at three different angles to the field (the normal to the PGS sheets always lies in the x-z plane). A field perturbation following a conventional dipolar field pattern (D), is evident when the normal to the stack is nearly parallel (2°) or perpendicular (90°) to the field, with a greater amplitude in the parallel case. At the intermediate angle (64°) the field contribution (E) from M_x is evident from the rotation of the field pattern. Linear regression of the amplitude of (D) against $\sin^2\Theta$ and of (E) against sin2Θ gave R^2 values > 0.99, indicating that the field variation follows that expected from theory. Using Eq. [2], the slopes of these plots gave values of χ_A/χ_I = -260±10/-135±10ppm (D) and χ_A = -287±20 ppm (E) for the 25 μm PGS, where the value of χ_I is measured relative to the susceptibility of agar. *Experiment 2* Figure 2 shows a portion of the field map acquired with the three PGS covered tubes oriented perpendicular to the field. A negative offset whose magnitude increases with decreasing tube diameter, as predicted by the full theory, is evident in this map. Figure 3 shows plots of the variation of the average field inside the three tubes as a function of $\sin^2\Theta$, where Θ is the angle between the field and the axis of the tube. The good straight line indicates that the field perturbation follows the theoretical prediction, while the slopes yielded a value of χ_A = -250±20 ppm for the 25 μm thick PGS. ***Experiment 3*** Figure 3 also shows that the amplitude of the field varying as cos 2ϕ outside the PGS-covered tube is proportional to $\sin^2\Theta$ and that there is no evidence of any significant variation with $\sin^4\Theta$ as has previously been suggested (based on modelling the anisotropy using an orientation dependent χ_I).

Discussion These results confirm the importance of including the effects of induced magnetisation that is orthogonal to the applied field, when calculating the field perturbation due to anisotropic magnetic material. Experimental measurements on a hollow cylinder formed from anisotropic material with radially-oriented principal axis (which is a model for the myelin sheath) confirm that the amplitude of the spatially varying field outside the cylinder vary purely as $\sin^2\Theta$, where Θ is the angle between the field and the cylinder axis, and demonstrate that a spatially uniform orientation-dependent field offset is produced inside the cylinder [5]. **References** [1] Lee *et al.* NIMG 57, 225, 2011. [2] Lee *et al.* PNAS 107, 5130, 2010. [3] Liu MRM. 63, 1471, 2010. [4] Li *et al.* NIMG. 59, 2088, 2012. [5] Wharton and Bowtell. PNAS. doi:10.1073/pnas.1211075109 2012.