Model-Based MR Parameter Mapping with Sparsity Constraint

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Introduction: MR parameter mapping is a valuable tool for tissue characterization. However, its practical utility has been limited by long data acquisition times. To address this problem, a number of approaches have been proposed to enable parameter mapping from undersampled data. One approach is to reconstruct the parameter-weighted image sequence from undersampled data using various constraints (e.g., sparsity constraint [1] or partial separability constraint [2]), followed by parameter estimation from the reconstructed image sequence. Several successful examples are described in [3]-[6]. Another approach is to directly estimate the parameter map from the undersampled **k**-space data, bypassing the image reconstruction step completely [7]-[9]. In this work, we propose a new method based on the second approach but allows sparsity constraint to be effectively used for improved parameter estimation. Some representative results from T_2 brain mapping are shown to illustrate the performance of the proposed method.

Method: Here we use T_2 mapping as an example to describe the proposed method, although it is generally applicable to other types of parameter mapping. For multi-echo T_2 mapping, we assume that the T_2 weighted images $I_m(\mathbf{x})$ can be expressed as $I_m(\mathbf{x}) = \rho(\mathbf{x})\exp(-R_2(\mathbf{x})T_{Em})$, where $\rho(\mathbf{x})$ represents the distribution of the spin density, $R_2(\mathbf{x})$ represents the distribution of the transverse relaxation rate, and T_{Em} is the m-th echo time. $I_m(\mathbf{x})$ is related to **k**-space data by $d_m(\mathbf{k}) = \int I_m(\mathbf{x})\exp(-2\pi\mathbf{k}\cdot\mathbf{x})d\mathbf{x} + n_m(\mathbf{k})$. After proper discretization, we have $\mathbf{d}_m = \mathbf{F}_m\mathbf{I}_m + \mathbf{n}_m$, where $\mathbf{F}_m$ denotes the Fourier encoding matrix, $\mathbf{I}_m = \mathbf{\Phi}_m(\mathbf{R}_2)\boldsymbol{\rho}$, $\mathbf{\Phi}_m(\mathbf{R}_2)$ is a diagonal matrix with the diagonal entry $[\mathbf{\Phi}_m]_{\mathbf{n},\mathbf{n}} = \exp(-R_2(\mathbf{x_n})T_{Em})$ and $R_2(\mathbf{x_n})$ represents the relaxation rate at the nth voxel. Given this observation model and assume that $\mathbf{n}_m$ are mutually independent Gaussian noise vectors, we can directly determine $\mathbf{R}_2$ from the measured data $\mathbf{d}_m$ using a penalized maximum likelihood (PML) formulation as follows:

$$\{\hat{\boldsymbol{\rho}},\ \hat{\mathbf{R}}_2\} = \arg\min_{\boldsymbol{\rho},\mathbf{R}_2} \sum_{m=1}^{M} \left\| \mathbf{d}_m - \mathbf{F}_m \Phi_m(\mathbf{R}_2)\boldsymbol{\rho} \right\|_2^2 + \lambda\Psi(\mathbf{R}_2)\ , \qquad (1)$$

where $\Psi(\cdot)$ is a regularization functional and λ is the regularization parameter. It is well known that in parameter mapping, the values of $\mathbf{R}_2$ are tissue-dependent. Since the number of tissue type is relatively small as compared to the number of image voxels, we can apply a sparsity constraint to $\mathbf{R}_2$ with an appropriate sparsifying transform. Since directly enforcing sparsity constraint through the ℓ_0 norm can be practically difficult, a total variation regularization is used to enforce sparsity in the finite difference domain, i.e., $\Psi(\mathbf{R}_2) = \mathrm{TV}(\mathbf{R}_2) = \sum_{n=1}^{N} |\mathbf{D}_\mathrm{h}^{(n)}\mathbf{R}_2| + |\mathbf{D}_\mathrm{v}^{(n)}\mathbf{R}_2|$. We propose an alternating minimization algorithm based on half-quadratic regularization and Quasi-Newton methods to solve (1). Specifically, with the Huber function approximation [10], it can be shown that Eq. (1) can be converted into

$$\min_{\boldsymbol{\rho},\mathbf{R}_2,\mathbf{g}_h,\mathbf{g}_v} \sum_{m=1}^{M} \left\| \mathbf{d}_m - \mathbf{F}_m \Phi_m(\mathbf{R}_2)\boldsymbol{\rho} \right\|_2^2 + \frac{\lambda}{2\beta}\left\| \mathbf{g}_\mathrm{h} - \mathbf{D}_\mathrm{h}\mathbf{R}_2 \right\|_2^2 + \lambda\left\|\mathbf{g}_\mathrm{h}\right\|_1 + \frac{\lambda}{2\beta}\left\| \mathbf{g}_\mathrm{v} - \mathbf{D}_\mathrm{v}\mathbf{R}_2 \right\|_2^2 + \lambda\left\|\mathbf{g}_\mathrm{v}\right\|_1\ , \qquad (2)$$

where $\mathbf{g}_\mathrm{h}$ and $\mathbf{g}_v$ are two auxiliary variables. Alternating minimization procedures for solving (2) are: for a fixed $\mathbf{R}_2$ and $\boldsymbol{\rho}$, we update $\mathbf{g}_h$ and $\mathbf{g}_v$ by a soft-thresholding operation; for a fixed $\mathbf{g}_h$ and $\mathbf{g}_v$, we update $\mathbf{R}_2$ and $\boldsymbol{\rho}$ by a Quasi-Newton Broyden-Flecher-Goldfarb-Shannon algorithm [11]. To improve the accuracy of Huber function approximation, a continuation procedure can be applied as in [10].

N_{eq}	CS	Proposed
3	12.0%	**5.2%**
4	8.4 %	**4.5%**
5	5.9 %	**4.2%**

Table 1: The NRMSE of reconstructed R_2 maps at three undersampling ratios.

Results: We applied the proposed method to a T_2 brain imaging experiment using a numerical phantom [12], which simulates a multi-echo spin echo acquisition with a total number of 14 echoes and 17.2 ms echo spacing. We undersampled the **k**-space at the ratios of 4.6, 3.5 and 2.8. The corresponding acquisition times are equivalent to acquiring 3, 4, 5 fully sampled image frames (denoted as N_{eq}). We added complex white Gaussian noise to the **k**-space data such that the ratio of the signal (in a region of the grey matter) to the noise standard deviation is 20dB. We compared the proposed method with a dictionary learning based compressed sensing reconstruction method (referred to as CS) [3], which only takes into account the temporal relaxation process. The normalized root mean square error (NRMSE) for the reconstructed $\mathbf{R}_2$ map is shown in Table 1. The reconstructed $\mathbf{R}_2$ maps for N_{eq}= 3 and 5 are shown in Fig. 1. As can be seen in this figure, when less data were acquired (i.e., N_{eq} = 3), the CS reconstruction suffers from severe artifacts, although these artifacts significantly reduced when more data were acquired (i.e., N_{eq} = 5). In contrast, the proposed method produces high-quality parameter maps at both high and low undersampling ratios. The observations are consistent with the NRMSE shown in Table 1.

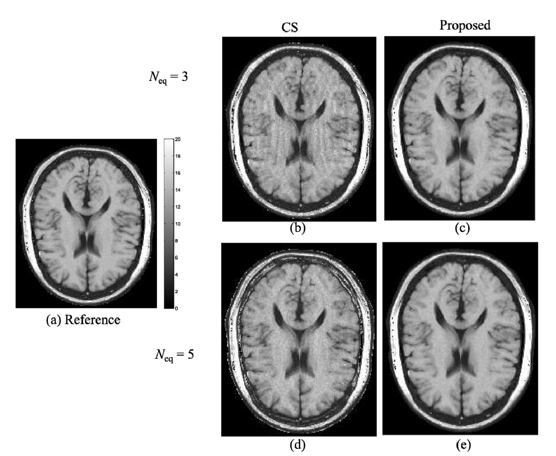

Fig. 1. (a) Reference R_2 estimated from fully sampled data; (b)-(c) Reconstructed R_2 maps using N_{eq} = 3; (d)-(e) Reconstructed R_2 maps using N_{eq} = 5.

Conclusion: We proposed a new method to directly reconstruct parameter maps from highly undersampled, noisy **k**-space data, utilizing an explicit signal model while imposing a sparsity constraint on the parameter maps. An algorithm was described to solve the underlying optimization problem. Representative results from a T_2 brain imaging example were also presented to illustrate the performance of the proposed method. The proposed method should prove useful for fast MR parameter mapping with sparse sampling.

Reference: [1] Lustig *et al*, *MRM*, 58:1182-1195, 2007. [2] Liang, *ISBI*, 988-991, 2007. [3] Doneva *et al*, *MRM*, 4:1114-1120, 2010. [4] Bilgic *et al*, *MRM*, 66:1601-1615, 2011. [5] Petzschner *et al, MRM, 66:706-716.* [6] Zhao *et al*, *ISMRM*, p. 2233, 2012. [7] Haldar *et al*, *ISBI*, 266-269, 2009. [8] Block *et al*, *IEEE TMI* 1759-1769, 2009. [9]Sumpf *et al*, *JMRI*, 420-428, 2011. [10] Zhao *et al*, *IEEE TMI*, 1809-1820, 2012. [11] Nocedal *et al*, Numerical Optimization, 2006. [12] Collins *et al, IEEE TMI, 463-468, 1998.*