Noniterative Closed Form Solution to Multipeak Proton-Density Fat Fraction Estimation

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TARGET AUDIENCE: Physicists, clinical radiologists and clinical researchers in the field of fat-water separation and fat fraction estimation.

PURPOSE: Show multipeak proton-density fat fraction (FF) can be estimated in a closed form without the need for an iterative algorithm.

METHODS: A popular gradient recalled echo (GRE) signal model for fat-water admixture incorporates a multipeak fat spectral model and assumes a common transverse relaxation rate for all proton species [1],

$$s(t) = \left(\rho_w e^{i f_w t} + \rho_f \sum_p c_p e^{i f_p t}\right) e^{(-R_2^* + i\psi)t} \qquad \text{[Eq. 1]}$$

Here, *s(t)* is the echo time-dependent fat-water complex signal, ρ_w and ρ_f proton-densities (PDs) of water and fat (unknown), R_2^* the common transverse relaxation rate (unknown), f_w the known frequency of the water peak, f_p the known frequency of the *p*-th fat peak, and c_p the known relative PD of the *p*-th peak such that $\sum_p c_p = 1$. The linear combination of the individual fat signals $\sum_p c_p e^{i f_p t}$ is referred to as the fat spectral model or basis function. The T_1-weighting term is omitted under the assumption that a sufficiently small flip angle is used.

Previous approaches to multipeak PD fat fraction (FF) estimation have used nonlinear curve fitting to iteratively find least-square estimates of the model parameters, including ρ_w and ρ_f. FF is calculated from the estimated PDs as $FF = \rho_f/(\rho_f + \rho_w)$ in this "nonlinear fit method". Traditionally, 6 echoes have been used for fitting, but emerging data suggest that fewer echoes (e.g. 3 or 4) may suffice [2].

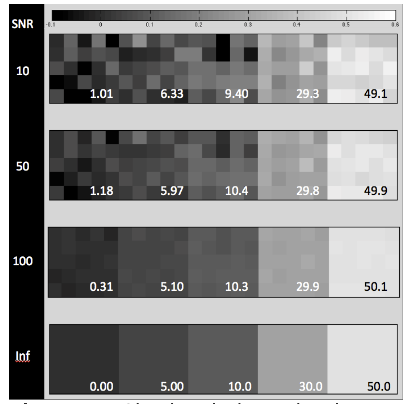

Figure 1: Simulated data showing FF estimation using the closed form method. FF estimation error is within 2% at all tested SNR.

In this report, we consider a special case where *s(t)* is sampled at a regular interval, Δt. After rearrangement of terms the sampled echoes s_n can be written as

$$s_n = s(n\Delta t) = k\rho_w \left(1 + \frac{\rho_f}{\rho_w} \sum_p c_p e^{i f_p' \Delta t}\right) e^{[-R_2^* i(\psi + f_w)]n\Delta t}$$

where $f_p' = f_w - f_p$ is the frequency of fat peaks relative to the water frequency. Let the fat signal basis function evaluated at $n\Delta t$, $B_n = \sum_p c_p e^{i f_p' n\Delta t}$ (known), and fat-water ratio, $r = \rho_f/\rho_w$ (unknown). Taking the first 3 consecutive echoes, the complex decay term cancels out by forming the ratio

$$s_1 s_3 / s_2 s_2 = (1 + rB_1)(1 + rB_3) / (1 + rB_2)(1 + rB_2)$$

which can then be rearranged as a quadratic function of *r*,

$$r^2\left(B_1 B_3 - A B_2^{\,2}\right) + r(B_1 + B_3 - 2AB_2) + (1 - A) = 0; \quad A = \frac{s_1 s_3}{s_2 s_2}$$

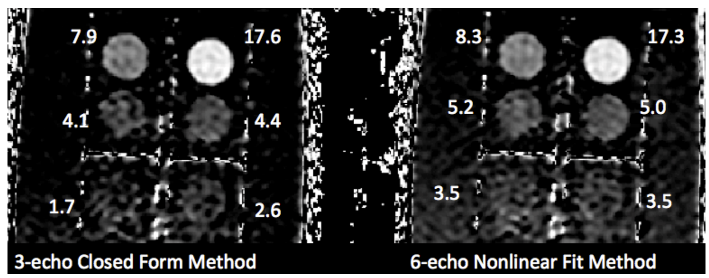

Figure 2: Comparison of 3-echo closed form vs. 6-echo nonlinear fit methods in an Intralipid phantom. Good agreement (within 2%) is seen between the two methods.

Therefore the fat-water ratio *r* is the root of a quadratic equation and can be solved in a closed form by invoking the quadratic equation. Fat fraction can be calculated from the real part of *r* by $FF = 1 - 1/[1 + 1/Re(r)]$. We call this the "3-echo closed form method".

This closed form method was validated in simulated and phantom datasets. Simulated data of FF = 0, 5, 10, 30, and 50% were generated in 5x5 pixel grids using Eq1 and R_2^* of 50Hz (i.e. T_2^* 20ms) and mixed with increasing complex Gaussian noise at SNR of 100, 5, and 10. A fat-water phantom was constructed using serially diluted aliquots of 20%-by-weight soybean oil emulsion (*Intralipid*, Baxter Healthcare, IL), i.e. 20, 10, 5, 2.5, and 1.25%. The phantom was imaged using a 1.5T whole-body system using a multiecho 3D GRE sequence, TE = 2.3, 4.6, 6.9, 9.2, 11.5, and 13.8ms, TR 15ms, and flip angle 2^o. The FF map was reconstructed using the closed form method and compared to the true FF values (simulated data) and to the FF map of the nonlinear fit method (phantom data).

RESULTS: In the simulated dataset with increasing noise, the closed form method had high FF estimation accuracy even at relatively low SNR of 10 (**Fig 1**), confirming the method's validity and suggesting stability in reasonably noisy data. In the phantom dataset, the closed form method performed comparably to the 6-echo nonlinear fit method (**Fig 2**), suggesting its potential role as a viable alternative to the nonlinear fit method.

DISCUSSION: The advantage of the proposed closed-form FF estimation method is that it avoids many of the technical drawbacks of the conventional nonlinear least square curve fit methods such as: (1) iterative computation, (2) dependency on the choice of initial estimates, (3) convergence at local minima, and (4) non-convergence, thus it is computationally fast and robust. The limitation of this method is the requirement of evenly-spaced echo-times.

CONCLUSION: Closed form estimation of multipeak proton-density FF allows for fast and robust non-iterative reconstruction of FF maps. It compares favorably to the conventional nonlinear least-square fitting methods.

References: [1] Yu, et al. Magn Reson Med. 2008 November; 60(5): 1122–113.; [2] Levin et al. to be presented at RSNA Annual Meeting, 2012;