A Geometric Interpretation of Water-Fat Identification in Two-Point Dixon Imaging without Phase Correction

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Introduction

The use of a multiple-spectral-peak fat model [1] breaks the symmetry between the two possible sets of solutions for water and fat in two-point Dixon imaging. Further, one of the two solutions may be rendered unphysical at certain echo time combinations, leaving only one feasible solution without resorting to phase correction [2]. A clear insight into when and how this happens would help guide data acquisition and post-processing. Here, we present a geometric interpretation of the problem and examine the impact of noise on the robustness of this technique in performing fat-water separation at different echo times.

Methods

We assume signals given by $S_n = (W + c_n F)p_n$, where $n = 1,2$; c_n is a complex coefficient that contains both the phase and amplitude of fat relative to water; and p_n is a background phasor. The solution (W,F) is an intersection point of the two ellipses given by the equations $W^2 + 2Re(c_n)FW + |c_n|^2F^2 - |S_n|^2 = 0$. These ellipses are centered at the origin and tilted with respect to the W axis by an angle ψ_n given by

$$\psi_n = \frac{1}{2}\cot^{-1}\left(\frac{1-|c_n|^2}{Re(c_n)}\right).$$

In the single-peak fat model, $|c_n|^2 = 1$, and therefore $\psi_n = 45°$. Consequently, both ellipses are symmetric across the line $W = F$, and two solutions always exist in the first quadrant (Fig. 1a). In the multi-peak model, however, $|c_n|^2 < 1$. Thus, the ellipses are no longer symmetric across $W = F$, and for certain echo time combinations the incorrect solution moves into an unphysical region and can be discarded (Fig. 1b).

The presence of noise, however, broadens the ellipses and may render the selection of the correct solution based on the non-negativity of W and F unreliable. Accurate identification of W and F depends on the noise level and on how far the incorrect solution deviates from the region of physically allowable solutions. In general, selection will be more reliable at high SNR (Fig. 1c) than at low SNR (Fig. 1d).

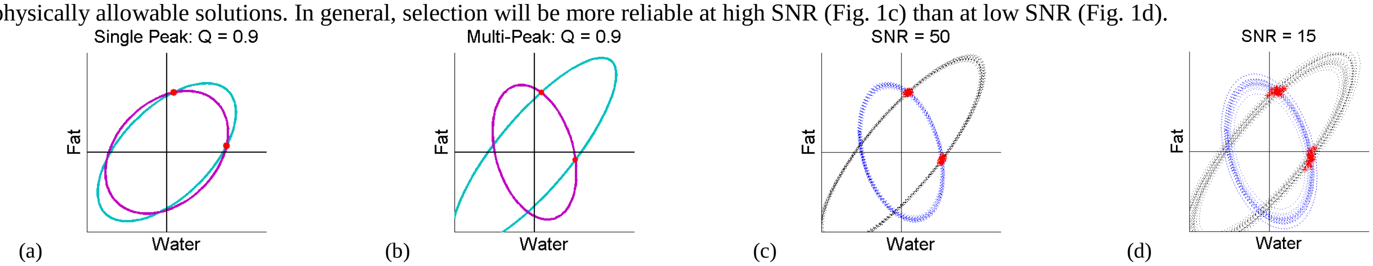
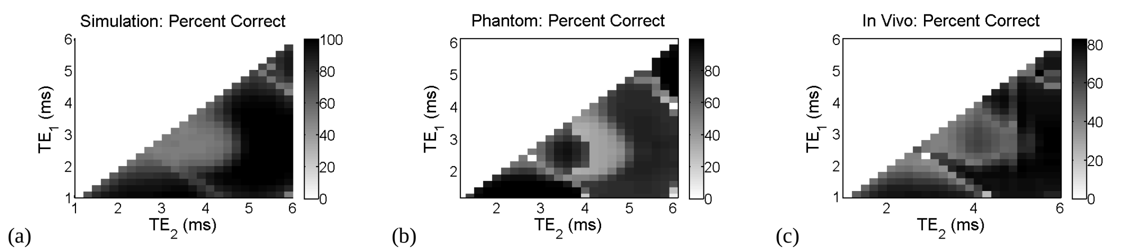


Fig. 1. Possible (W, F) solutions for a true fat fraction of $Q = 0.9$ with echo times $TE_1 = 3$ and $TE_2 = 6$ ms. (a) In the single peak model, both possible solutions are physical. (b) In the multi-peak model, the incorrect solution has a negative F value and can be discarded. (c) At $SNR = 50$, the solutions remain unambiguous. (d) At $SNR = 15$, some of the correct solutions appear unphysical, while some of the incorrect solutions appear physical.

Results

Fig. 2. Percentage of correctly identified pixels. (a) Simulation with $SNR = 50$. (b) Oil-water phantom. (c) In vivo (legs).



To quantify the impact of noise, we calculated the percentage of pixels correctly identified by the above method for simulated data with various SNR values, as well as for oil-water phantom data and in vivo data acquired at various TE1-TE2 combinations on a 1.5 T whole-body scanner. In simulation, a multipeak fat model [1] was used, whereas for the phantom and in vivo data, we calibrated the fat signal model by directly measuring phase and amplitude of fat at different echo times. The phantom and in vivo results (Fig. 2 and 3) show that nearly 100% of pixels are correctly identified for certain ranges of TE1/TE2, while for other TE1/TE2 combinations, the percentage is substantially lower.

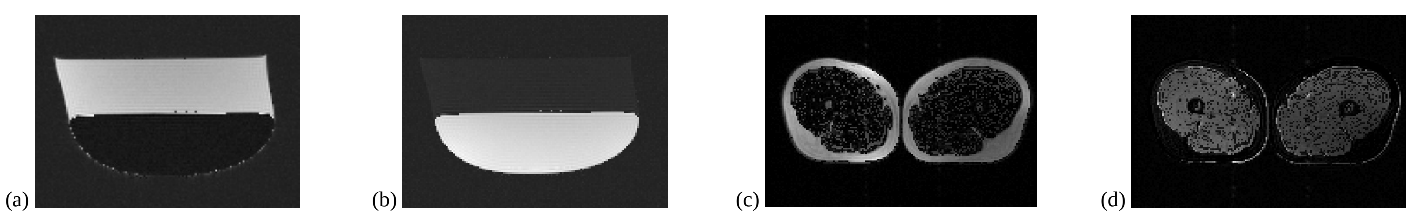


Fig. 3. Separated images without further processing. (a) Fat and (b) water for phantom at $TE_1 = 1$ ms, $TE_2 = 2$ ms. (c) Fat and (d) water for in vivo scans at $TE_1 = 3$ and $TE_2 = 6$ ms at 1.5 T. More swapping occurs in the water region, where SNR is much lower (approximately 25).

Conclusion

The presented geometric interpretation helps with understanding and visualizing the factors that may affect the selection of the correct solution in two-point Dixon imaging with flexible echo times. Our results show that determining the correct solution without phase correction is possible only for certain echo time combinations and at high SNR. With an increasing noise level or deviation from the signal model, this selection method becomes less reliable. However, this method may be used to obtain a first-pass solution to aid subsequent processing by phase correction.

References

1. Ren J, et al. J Lipid Res 2008; 49:2055-2062. 2. Eggers H. Proc Intl Soc Mag Reson Med 20 (2012).