Optimal PLD design and maximum likelihood CBF estimation for dynamic PCASL with Rician noise

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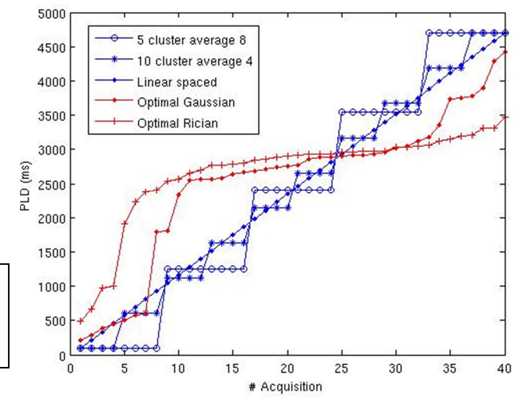
Purpose: Dynamic ASL imaging can provide important information (such as CBF and arterial transit time) for the care of stroke and brain tumor patients. Currently ASL requires significant signal averaging to improve image quality and parametric accuracy. By designing multiple post label delays (PLDs) instead of averaging, we can obtain additional perfusion parameters and achieve more accurate estimation [1, 2]. The design of PLD times and the design of estimators are typically based on a Gaussian noise assumption. However, in rapid dynamic multi-PLD ASL, the SNR is very low and we cannot treat magnitude Rician noise as Gaussian. A Gaussian noise model results in biased estimation with the least squares (LS) method and sub-optimal PLD design. Here, we present the optimal multi-PLD design of a PCASL experiment for CBF estimation with Rician noise and an unbiased maximum likelihood estimator assuming Rician noise.

Methods: In experiment design, the theoretical accuracy of the estimate is constrained by the Cramér–Rao lower bound (CRLB):

$$E[(\hat{x}-x)^2 \mid x] \geq \frac{1}{E[(\frac{\partial \ln p(y\mid x)}{\partial x})^2]}$$



PLD sampling pattern design

Even if the CRLB is not achievable, we still want to maximize the Fisher matrix (the denominator) so as to minimize the variance of our estimated parameters. To calculate the expectation above, any noise model can be used. MRI acquisition results in complex Gaussian noise, and after the magnitude operation it has a Rician distribution. S is the acquired noisy signal, ΔM is the expected ideal perfusion signal, I_0 is 0^{th} order modified Bessel function of the first kind.

$$S(t) = \Delta M(t) + n$$

$$P(S) = \frac{S}{\sigma^2} e^{-\frac{S^2+\Delta M^2}{2\sigma^2}} I_0(\frac{S\Delta M}{\sigma^2})$$

$$\frac{1}{\sigma^2}\sum_{k=1}^{N}\frac{\partial \Delta M(t_k)}{\partial f}\left[S(t_k)\frac{I_1\left(\frac{S(t_k)\Delta M(t_k)}{\sigma^2}\right)}{I_0\left(\frac{S(t_k)\Delta M(t_k)}{\sigma^2}\right)} - \Delta M(t_k)\right] = 0$$

In the case of high SNR, Rician noise can be approximated by Gaussian noise, and LS estimation yields an efficient estimator. However, when the SNR is low, LS results in biased estimation and the more general method of maximum likelihood estimation (MLE) is needed for an unbiased estimate. MLE can achieve the CRLB when the data set is large enough, regardless of the noise model.

All simulations were performed using MATLAB 2012a. 40 simulated ASL acquisitions were done with the following different PLD designs, as illustrated in the figure above: (1) 5 cluster PLDs with 8 averages; (2) 10 cluster PLDs with 4 averages; (3) Linearly spaced PLDs; (4) Optimal PLD design based on Gaussian noise; and (5) Optimal PLD design based on Rician noise. All optimal PLD designs assume both Δt and CBF are unknown, and design to minimize CBF variance. With each PLD design and low SNR (σ = $1/300 * M_{0,blood}$), data was simulated, then noise was added and CBF was estimated. Each case was repeated 10000 times to verify the statistical performance of PLD design and estimator. The real value of CBF is 72ml/100g/min. Other common assumptions are made for the perfusion model: tissue T1=1300ms, blood T_{1b}=1600ms, labeling efficiency α=0.9, λ=0.9, bolus duration τ=2000ms, arterial transit time Δt=700ms. The classic single-compartment Buxton PCASL model is used here.

	Gaussian noise	Rician noise		Normalized Theoretical Variance	
	LS	LS	Rician MLE	Gaussian	Rician
5 Cluster PLDs (average 8)	72.0569±5.1695	78.2425±5.0122	72.0548±6.0945	100%	108.83%
10 Cluster PLDs (average 4)	72.0298±4.8615	77.5247±4.6611	71.9385±5.5472	89.07%	96.06%
Linear PLDs	72.0263±4.7262	77.5262±4.5309	72.0273±5.4101	76.01%	82.51%
Optimal Gaussian Noise PLDs	71.9816±3.7740	75.2608±3.5920	72.0115±3.9428	49.93%	51.29%
Optimal Rician Noise PLDs	71.9520±3.4251	75.2191±3.3790	72.0486±3.6407	42.98%	44.02%

Results and Discussion: The results are given in the table above. (1) In both theoretical Fisher Information and simulation variance, we can reduce the CBF variance by about 50% by optimal PLD design. Therefore, optimal PLD design instead of averaging could significantly improve ASL parameter mapping. (2) With low SNR and Rician noise, LS results in biased estimation and Rican MLE gives the correct value. We should also note that although the Rician MLE is unbiased, it has slightly higher variance than LS. So, an interesting question arises: do we prefer a biased estimate with a lower variance or an unbiased estimate? (3) There is little improvement by optimal Rician design as compared to optimal Gaussian design, but both are significantly better than averaging.

Conclusion: Optimized PLD design can reduce CBF estimation variance significantly and MLE can provide unbiased estimation in the typical Rician noise situation.

References: 1. Xie J, Gallichan D, Gunn R, et al. Optimal design of pulsed arterial spin labeling MRI experiments. MRM. 2008;59: 826-834. 2. Zhao L, Meyer C. Optimal kinetic PASL design and CBF estimation with low SNR and Rician noise. ISMRM, 2012;3494.