Non-Negative Spherical Deconvolution for Fiber Orientation Distribution Estimation

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Introduction: In diffusion MRI, Spherical Deconvolution (SD) was proposed to estimate the fiber Orientation Distribution Function (fODF) $\Phi(\mathbf{u})$ based on spherical deconvolution using a single-fiber response function $H(\mathbf{u})$[1,2]. The peaks or the shape of fODFs can be used to infer local fiber directions. Constrained Spherical Deconvolution (CSD) [1], which takes into consideration the non-negative of the fODF, is the most widely used method among SD variants. Although CSD is capable of accurately determining fiber directions, it is susceptible to false positive peaks especially in the regions with low anisotropy. This is a common drawback of all existing SD-based methods. Moreover, in practice the fODF estimated using CSD still has significant negative values. We propose a method called Non-Negative Spherical Deconvolution (NNSD) to solve the above two problems. Based on a Riemannian framework of ODFs [3] and Square Root Parameterized Estimation for non-negative definite Ensemble Average Propagator [4], NNSD is formulated such that the non-negativity of the fODF is guaranteed with largely reduced false positive peaks.

Methods: We represent both the fODF and the signal from a single fiber along the z-axis (i.e., the kernel) using Spherical Harmonic (SH) basis with the maximal order L, i.e. $\Phi(\mathbf{u}) = \sum_{l,m}^{L} c_{lm} Y_l^m(\mathbf{u})$ and $H(\mathbf{u}) = \sum_{l,m}^{L} h_{lm} Y_l^m(\mathbf{u})$, respectively, where $\mathbf{u}$ is a unit vector and $Y_l^m(\cdot)$ is the SH basis with order l and degree m. Based on the theoretical results in [1,2], the MR signal attenuation is reconstructed using the spherical convolution between the fODF and the kernel as $E(\mathbf{u}) = \sum_{l,m}^{L} \sqrt{\frac{4\pi}{2l+1}} c_{lm} h_{lm} Y_l^m(\mathbf{u})$. In NNSD, we estimate instead the **square root** of the fODF [3,4], i.e. $\Phi(\mathbf{u}) = \left(\sum_{l,m}^{L} c_{lm} Y_l^m(\mathbf{u})\right)^2$. It can be proved that in this case the convolved signal is $E(\mathbf{u}) = \sum_{\alpha,\beta}^{2L} \sum_{l,m}^{L} \sum_{l',m'}^{L} \sqrt{\frac{4\pi}{2l+1}} c_{lm} c_{l'm'} Q_{ll'\alpha}^{mm'\beta} h_{\alpha\beta} Y_\alpha^\beta(\mathbf{u}) = \boldsymbol{c}^T \mathbf{K}(\mathbf{u}) \boldsymbol{c}$, where $Q_{ll'\alpha}^{mm'\beta} = \int_{S^2} Y_l^m(\mathbf{u}) Y_{l'}^{m'}(\mathbf{u}) Y_\alpha^\beta(\mathbf{u})\, d\mathbf{u}$ is a mathematical constant, $\boldsymbol{c} = [c_{00}, \dots, c_{LL}]^{\mathbf{T}}$ is the unknown coefficient vector, and $\mathbf{K}(\mathbf{u})$ for each $\mathbf{u}$ is a fixed symmetric matrix, i.e. $\mathbf{K}_{ll'}^{mm'}(\mathbf{u}) = \sum_{\alpha,\beta}^{2L} \sqrt{\frac{4\pi}{2l+1}} h_{\alpha\beta} Q_{ll'\alpha}^{mm'\beta} Y_\alpha^\beta(\mathbf{u})$. We then estimate the parameter $\boldsymbol{c}$ from the measured diffusion signal vector $\boldsymbol{E} = (E_1, \dots, E_N)^{\mathbf{T}}$ by minimizing

$$J(\boldsymbol{c}) = \sum_{i=1}^{N} (\mathbf{c}^T \mathbf{K}(\mathbf{u}) \mathbf{c} - E_i)^2 + \lambda \mathbf{c}^T \mathbf{L} \mathbf{c} \qquad \textbf{s.t. } \|\mathbf{c}\| = 1$$

where $\mathbf{L}$ is the Laplace-Beltrami regularization diagonal matrix with element $\mathbf{L_{lm}} = l^2(l+1)^2$. Since fODF is a probability density function and SH is an orthonormal basis, we have the constraint $\|\mathbf{c}\| = 1$ [3,4]. Riemannian gradient descent on the sphere is performed to minimize the above cost function [4]. $\mathbf{c} = (1,0,\dots,0)^{\mathbf{T}}$, indicating isotropic fODF, is used for initialization. The fODF is recovered by squaring the square root of the fODF naturally resulting in non-negativity of the fODF.

Experiments and Results: The proposed NNSD method was validated using both synthetic data and real data. The synthetic DWI data was generated based on a mixture of tensors with eigenvalues $[1.7,03,0.3] \times 10^{-3} \mathrm{mm}^2/\mathrm{s}$, with a *b*-value of $1500 \mathrm{s}/\mathrm{mm}^2$, and with 81 directions non-collinear gradient-sensitized directions. This data was perturbed with Rician noise with a low SNR=10.The SNR is defined as the ratio between the standard deviation of the complex Gaussian noise and the baseline signal without diffusion weighting. NNSD were compared with the state-of-the-art CSD using the same fiber kernel estimated from the voxels with FA larger than 0.8. The CSD and the kernel estimation implemented by their authors in MRtrix (http://www.brain.org.au/software/mrtrix/) were used. For fair comparison, we chose for both methods a maximum order of 6 for the SH basis. We intentionally set $\lambda = 0$ so that NNSD can be compared with CSD with no smoothness regularization. The results are shown in the left two subfigures of Fig.1. The intensities of the background image represent the Generalized Fractional Anisotropy (GFA) [5] values calculated from the fODFs. The fODF glyphs were directionally colored. It can be observed that in both single-fiber and crossing regions, NNSD and CSD both can correctly detect the local fiber orientations. The fODFs computed by NNSD however do not contain any negative values, as opposed to CSD. In more isotropic regions, the fODFs given by NNSD have significantly lower GFA than those by CSD, signifying the fact that NNSD removes a significant amount of false positive peaks. Evaluation was also performed using real human data with a *b*-value of $2000\, \mathrm{s}/\mathrm{mm}^2$, 120 gradient directions, 2mm isotropic voxel dimensions, TR/TE=124,000ms/116ms. We used the same parameters as those in the synthetic data for CSD and NNSD. The results are shown in the right two subfigures in Fig.1.CSD results in a significant amount of false positive peaks, which is especially evident in the regions with low anisotropy, as indicated by the high background GFA values. NNSD obtains a very clean fODF field with similar peaks as detected by CSD in the high anisotropic regions and more isotropic fODFs in the isotropic regions, such as CSF and grey matter.

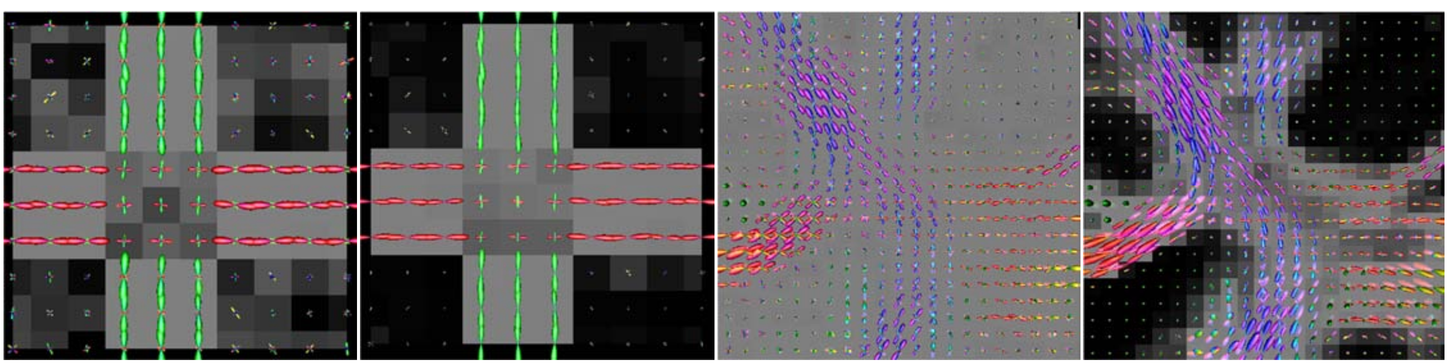

Fig. 1. CSD and NNSD results for synthetic data (two left panels) and real data (two right panels). Min-max normalization was not performed.

Conclusion: We have proposed a novel Spherical Deconvolution (SD) method called Non-Negative SD (NNSD) to overcome the two main limitations of existing fODF estimation methods. First, NNSD guarantees that the estimated fODFs are non-negative across the whole continuous spherical domain. Second, NNSD reduces significantly the false positive peaks that are prevalent in methods like CSD [1]. The clean fODF field given by NNSD will help reducespurious fiber tracts in tractography algorithms.

References: [1]. Tournier, J., et al., Neuroimage, 34(4), 1459-1472, 2007. [2]. Anderson, A., et al., Magnetic Resonance in Medicine, 54(5), 1194-1206, 2005. [3] Cheng, J., et al., MICCAI 2009, LNCS 5761, 911-918. [4] Cheng, J. et al., MICCAI 2012, LNCS, 6892, 98-106. [5] Tuch, D. S., Magnetic Resonance in Medicine, 52, 1358-1372.