A High-Speed Diffusion-Weighted MRI Simulator with Bloch-Torrey Equation

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Introduction

A diffusion-weighted MR Imaging (DWI) simulator based on Bloch-Torrey equation has been developed recently [1] and was applied to generate DWI images for very small models, such as cells [2]. However, to our knowledge DWI images for human brain size models have not been generated because large computational power is required. The purpose of this study is to develop a DWI simulator, which can generate the images for such larger models in a reasonable time. In order to shorten the computation time, we proposed three approaches based on the principle of MRI to increase simulation efficiency and succeeded in improving the efficiency approximately 140,000 times. As a result, simulation time for an adult human brain size model reduced to less than one hour.

Methods

Finite-difference diffusion simulation: Simulation was performed only at discrete spatial locations using iteration solution of Bloch-Torrey equation with rotation matrices and finite-difference equations. Simulation time increases according to the number of isochromats to be calculated and its accuracy depends on the gap interval of the isochromats. Therefore, an accurate simulation for human brain size models needs a lot of isochromats to be calculated, resulting a very long simulation time. In order to make the error less than 3 %, gap interval of the isochromats should be less than 2 μm (for diffusion coefficient $D=3.0\times10^{-3}$ mm^2/s). We propose the following three approaches to increase simulation efficiency by two assumptions (a) and (b).

(a): Characteristic values (density, two relaxation times and diffusion coefficient) of the isochromats placed in the unit area of the numerical model are same for all isochromats. It is possible to combine several adjacent unit areas with same characteristic values. This combined area is labeled the uniform area.

(b): Although the diffusion occurs in all directions, MRI signal attenuations due to diffusion are only enhanced by the field gradient in the direction of MPG.

Three acceleration approaches: (I) Isochromats are divided into two categories (Fig. 1a), the border isochromats located around the borders of the uniform areas, and the inner isochromats located outside the border areas. As the pulse sequence evolutes, the magnitudes of the border isochromats are varied by the diffusion among the surrounding isochromats. However, the magnitudes of the inner isochromats are equal for all isochromats except for the location-dependent phases induced by MPG. It means that the calculations of the inner isochromats can be easily evaluated by extrapolating the values of the border isochromats. This simplified calculation allows an efficient calculation of the inner isochromats for only the signal acquisition period (Fig. 1a). In contrast, the calculations of the border isochromats are necessary for the entire time of the pulse sequence. The magnitudes of the initial distributions (Fig. 1b) of the border isochromats vary according to the evolution of the pulse sequence. At the end of the border, the diffusion term in Bloch-Torrey equation cannot be evaluated accurately, because the values for the inner isochromats are not calculated yet. Therefore, error areas are generated at both ends of the border isochromats (Fig. 1c). After eliminating the error area (56 μm for $D=3.0\times10^{-3}$ mm^2/s), the values of the inner isochromat $M(t)_1$ are extrapolated accurately by using the value of the end isochromat $M(t)_0$ of the flat area of the border isochromats (Fig. 1d). The calculations of the phase difference between adjacent isochromats are done by only rotating the phase depending on the amount of MPG.

(II) The gap intervals of the isochromats are set to vary according to the direction of MPG (Fig. 2). For example, when MPG is applied in the x-direction, gap intervals of the isochromats in this direction are set to 2 μm, and these of the y-direction are set to 1.6 mm in the simulation.

(III) Simulation is performed in parallel by using a 4-core multiprocessor (2.9 GHz Intel Core i7). The whole code of the simulator is written in C language.

Numerical model: An ellipsoidal model of 134×162 mm^2 is used. It consists of 8,484 unit areas of 1.6×1.6 mm^2. Magnitudes of all isochromats are set to 1 to enhance the effects due to the differences among the diffusion coefficients. Relaxation times and diffusion coefficients are set to different values for each tissue. GE sequence is used with MPG of b=1,001s/mm^2 and TE of 57.5 ms.

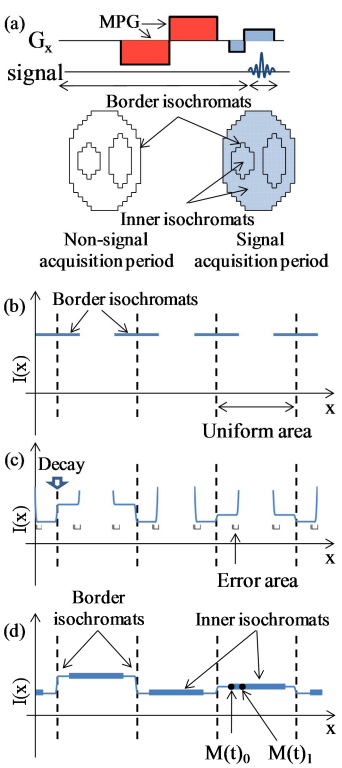

Fig. 1: Diagram of acceleration approach(I).

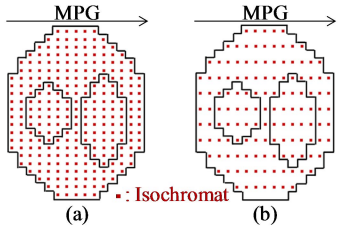

Fig. 2: (a)Distribution of isochromats without acceleration approach(II). (b)Distribution of isochromats with acceleration approach(II).

Results and Discussion

The simulation was performed by using all three approaches. Simulation results are shown in Fig. 3. Simulation time was 52 minutes and was 147,000 times shorter than without using these acceleration approaches. The errors were estimated by comparing the simulated values with the theoretical values calculated from the b-value and the diffusion coefficient. The results showed the errors were less than 0.2 %. Acceleration ratios for each approach were estimated: (I) 46 times, (II) 800 times, and (III) 4 times. The most effective approach was MPG-dependent gap interval modulation. This involves alignment of isochromats when the direction of MPG changes, but it seems very effective to save computation time dramatically.

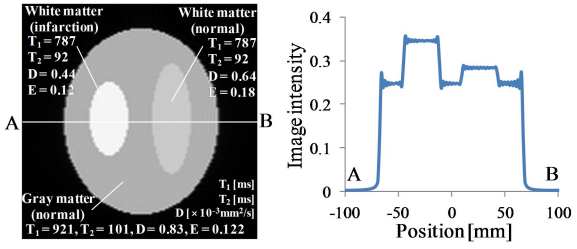

Fig. 3: Simulated image for a human brain size model (128 × 128 pixels) and intensity profile of AB. E is a computation error [%].

Conclusion

DWI high-speed simulator was developed and succeeded in generating the images for an adult human brain size model in a reasonable time. Three acceleration approaches were applied and were proven to be effective to increase efficiency of the simulation.

References

[1] Jochimsen TH et al. JMR 2006;180:29-38. [2] Imae T et al. 2011. Tokyo, Iryoukagakusha.