Low-Rank Basis Smoothing for the Denoising of Diffusion Weighted Images

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TARGET AUDIENCE: Diffusion Imaging (DI) investigators.

PURPOSE: The substantial signal attenuation in DI images for large b-values can affect accurate calculation of orientation distribution functions (odf) and fiber tracks. In addition, the low signal-to-noise (SNR) observed at large b-values hinders the performance of popular denoising methods like the LMMSE estimator [1] and the non-local means (NLM) filter [2]. In [3] the low-rank structure observed across several DWI images was exploited while imposing joint edge constraints within the images. In this work we demonstrate the benefits of basis smoothing within a low-rank DWI estimation framework. Our method significantly reduces the dependencies on noisy basis vectors while preserving root-mean-square error (RMSE) relative to low-noise data (computed by averaging multiple acquisitions).

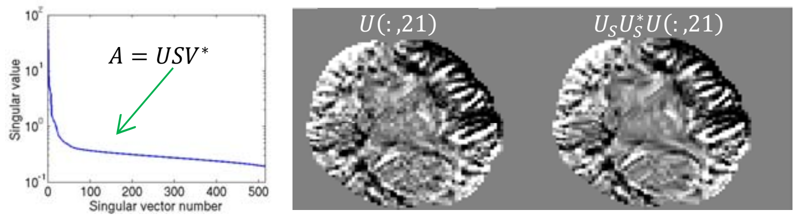

Fig. 1. Singular value decay for matrix A consisting of 515 direction DSI images. Noisy singular vector $U(:,21)$ image and corresponding smoothed basis vector.

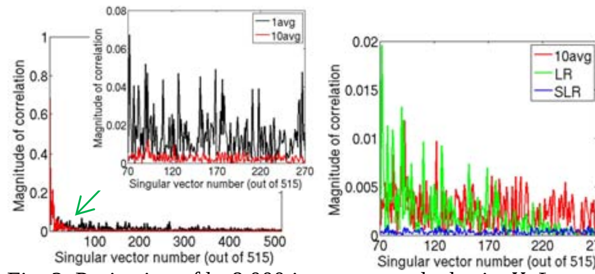

Fig. 2. Projection of b=8,000 images onto the basis U. Lower correlation with noisy singular vectors seen using SLR.

METHODS: In order to analyze the performance of our denoising strategy we have acquired a 515 direction healthy volunteer DSI data set with bmax = 8,000s/mm^2 at 2.3mm isotropic resolution on the 3T MGH-UCLA Skyra Connectome scanner. To obtain a low-noise data set for comparison we average across 10 acquisitions for 6 of the q-space points. The 515 DSI images are arranged into a 9216×515 matrix A, where each column of A is the vector form of a DWI image. The low-rank structure of A can be observed by computing the singular-value decomposition $A = USV^*$, see Fig. 1. By truncating all the singular values below a threshold σ_t we can form a regularized low-rank (LR) estimation $A_{LR} = U_t(S_t - \sigma_t/2)V_t^*$ (it is important to note that this is a simplified and computationally efficient version of [3]). The matrix U_t is a basis set for the dominant information contained in the 515 DSI images. However, as can be seen in Fig. 1 the basis vectors contained in U_t are still somewhat noisy due to the fact that they are estimated from noisy observations. Unlike the large b-value images themselves many of the singular vectors have relatively higher SNR and NLM algorithms can be employed effectively, see Fig. 1. Therefore, we project the low rank estimation onto a new basis set U_S that contains denoised versions of several singular vectors $A_{SLR} = U_S U_S^* U_t(S_t - \sigma_t/2)V_t^*$.

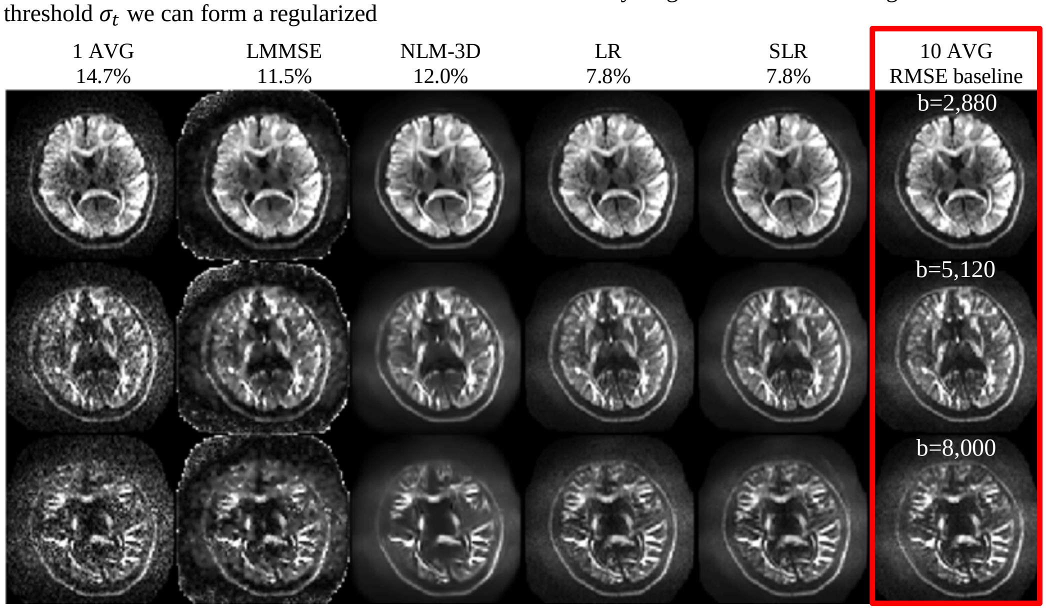

Fig. 3. DSI images (1 AVG) and several denoised images are shown for 3 q-space locations. Mean RMSE computed at 6 q-space points (against the 10 AVG images) is listed under the method name.

RESULTS: Fig. 2 shows the projection of b=8,000 images with 1-average or 10-averages (see Fig. 3) onto the basis U. The 10-average image has substantially less noise which is reflected in a damping on the correlations that correspond to the noisy basis vectors (higher singular value number and lower singular value). Fig. 2 shows the further reduction of the correlations for the LR and smoothed basis low-rank (SLR) images when projected onto U. Fig. 3 shows the RMSE when compared to the 10 average images at the 6 q-space points. For the LMMSE and NLM algorithms, publicly available implementations were used [4, 5]. Each algorithm was tuned to minimize the RMSE for these images.

DISCUSSION: The LMMSE and NLM algorithms were unable to denoise the large b-value images without corrupting key features that can be seen in the 10 average low-noise images. Both the LR and SLR approaches were able to maintain the critical features and subsequently had the smallest RMSE. In addition, the SLR algorithm reduced the dependency on noisy basis vectors to produce reduced noise speckling throughout the images.

CONCLUSION: We have introduced a framework for improving DWI image quality through low-rank basis smoothing. The method retains dominant image features while reducing dependencies on noisy basis vectors.

REFERENCES: [1] S. Aja-Fernandez, IEEE TMI'08; [2] N. Wiest-Daessle, MICCAI'08; [3] F. Lam, ISBI'12; [4] Matlab Central LMMSE toolbox; [5] Matlab Central 2D and 3D NLM toolbox; **SUPPORT:** NIBIB R00EB012107, R01EB006847, NCRR P41RR14075, NIH U01MH093765.