Improved L1-SPIRiT Reconstruction with a Phase Divergence Penalty for 3D Phase-Contrast Flow Measurements

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INTRODUCTION Time-resolved three-dimensional phase-contrast MR flow measurements (3D PC-MRI) suffer from long scan times and phase noise. Compressed sensing (CS) has been introduced to acquire undersampled volumetric MRI datasets in significantly reduced scan times. However, standard CS implementations regularize voxel magnitude while ignoring voxel phase. Recently Zhao *et al* have proposed separate regularization of magnitude and phase [1]. In their work, phase is regularized with a finite difference penalty, using either an L2 or L1 norm, depending on if jumps in phase are expected. While such an approach is viable, further improvements can be expected if prior knowledge from fundamental fluid mechanics are incorporated. It is the goal of the present work to integrate a phase divergence penalty [2,3] to enforce reconstruction of 3D velocity vector fields with negligible sinks or sources. The efficacy of the method is demonstrated using simulated and in-vivo data acquired in the aortic arch.

METHODS In this reconstruction approach the cost function as given in equation (1) is minimized, where **E** is the encoding matrix, **G** is a SPIRiT operator for parallel imaging [4], and **W** is a wavelet transform. In the case of a flow encoding scheme with a motion compensated null point, that phase is reconstructed first with R(ϕ) set to equation (2). This constraint enforces smoothness of the image phase with the L2 norm of a finite difference penalty between neighboring voxels **C**. The background phase is then subtracted from the velocity encoded phases, which then use R(ϕ) as in equation (3). Here the L1 norm is used to handle edges at the vessel wall, and the divergence is minimized as well as total variation. The total variation terms operate on exp(i ϕ) so that wraps do not affect the outcome. All phase regularizers are masked to remove phase from air voxels. The problem is solved by alternating between magnitude and phase updates, using nonlinear conjugate gradients for each.

$$f(\boldsymbol{m},\boldsymbol{\phi}) = \|\boldsymbol{y} - E\boldsymbol{m}e^{i\boldsymbol{\phi}}\|^2 + \|(G-I)\boldsymbol{m}e^{i\boldsymbol{\phi}}\|^2 + \lambda_1\|W\boldsymbol{m}\|_1 + \lambda_2 R(\phi) \quad (1)$$

$$R(\phi) = \|Ce^{i\boldsymbol{\phi}}\|^2 \quad (2)$$

$$R(\phi) = \|Ce^{i\boldsymbol{\phi}}\|_1 + \|div\boldsymbol{\phi}\|_1 \quad (3)$$

Table 1 Root mean squared error (RMSE) of different recons in the digital flow phantom. Values are scaled so that RMSE of the zero filled recon is 1.0

	RMSE magnitude	RMSE static velocities	RMSE vessel velocities
Zero Filled	1.00	1.00	1.00
L1-SPIRiT	0.473	0.536	0.476
L1 SPIRiT^{phase}	0.392	0.264	0.391

Data from a digital flow phantom and an in vivo chest measurement of the aorta were used to test the algorithm. A uniform linear background phase and noise (SNR=16) were added to the phantom before applying random undersampling (R=4). RMSE of magnitude, static voxel velocities, and flowing voxel velocities were compared for data reconstructed via zero-filling, standard CS (L1-SPIRiT), and the proposed method. The peak systolic phase of a fully sampled in-vivo dataset acquired in the aorta of a healthy volunteer (venc(x,y,z) 100,130,160cm/s; spatial resolution 1.43x1.43x1.43mm^3) was undersampled by a factor of 4 and reconstructed using L1-SPIRiT and the proposed L1-SPIRiT^{phase} method. The in vivo results were analyzed by comparing streamlines, flow, and peak velocity in the aorta (GTFlow, GyroTools, Zurich).

RESULTS With the proposed method, the RMSE in the phantom was significantly improved compared to the standard L1-SPIRiT approach (Table 1, Figure 1), while no reduction in through vessel flow measurements or peak velocity were observed (<1%). Streamlines in the in vivo data qualitatively appeared more laminar and had less noise-like jitter when using the L1-SPIRiT^{phase}(Figure 2). Peak velocities and flow values compared well with the reference from the fully sampled data along the superior regions of the aorta, but a slight decrease was noted for inferior regions.

DISCUSSION Incorporation of a divergence penalty into L1-SPIRiT shows promising results as a preliminary example of creating flow specific phase regularizers based on fluid mechanic constraints. It created more realistic flow profiles within vessels, significantly reduced velocity noise in static tissue, and improved magnitude images. In vivo streamline consistency was improved. Underestimation of velocities was noted in some regions and requires further investigation. Upon L1 regularization, velocity errors are primarily concentrated along the vessel walls where the gradients and divergence are highest, which could potentially be addressed by specifically modeling velocity fields in those areas. In further work, we will address the effectiveness in more complicated flow patterns, the problems underlying velocity underestimation, and using more advanced total variation penalties.

REFERENCES **[1]** Z. F. et al., "Separate Magnitude and Phase Regularization via Compressed Sensing," IEEE TMI, 2012. **[2]** Song, S. M. et al. "Noise reduction in three-dimensional phase-contrast MR velocity measurements." JMRI, 3(4), 587–96. 1993. **[3]** B. J. et al., "Construction of Divergence-Free Velocity Fields From Cine 3D Phase-Contrast Flow Measurements," MRM, 2012. **[4]** Lustig, M., Pauly, J. M. "SPIRiT: Iterative self-consistent parallel imaging reconstruction from arbitrary k-space." MRM, 2010.

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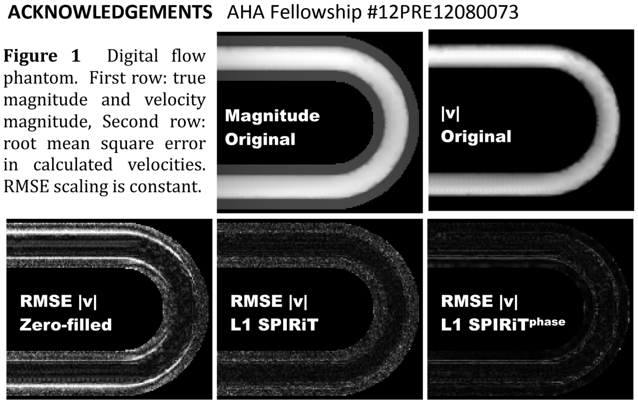
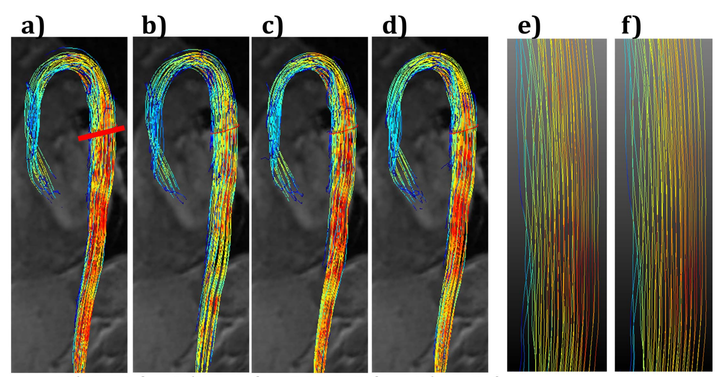


Figure 1 Digital flow phantom. First row: true magnitude and velocity magnitude, Second row: root mean square error in calculated velocities. RMSE scaling is constant.



	a)	b)	c)	d)
Flow:	**152 mL/s**	**150 mL/s**	**149 mL/s**	**151 mL/s**
Average Vel:	**86 cm/s**	**84 cm/s**	**85 cm/s**	**86 cm/s**
Max Vel:	**118 cm/s**	**112 cm/s**	**129 cm/s**	**116 cm/s**

Figure 2 Streamlines through the aorta from an in vivo dataset. a)-d) represent the full sampled data, zero-filled, L1-SPIRiT, and L1-SPIRiT^{phase} respectively. A measurement plane depicted as a red line in the original image, and the flow and velocity values through tha plane are displayed. e) and f) are zoomed in images of the streamlines from c) and d)