A robust spherical deconvolution method for the analysis of low SNR or low angular resolution diffusion data

J-Donald Tournier[1,2], Fernando Calamante[1,2], and Alan Connelly[1,2]

[1]*Advanced MRI development, The Florey Institute of Neuroscience and Mental Health, Melbourne, Victoria, Australia,* [2]*Department of Medicine, University of Melbourne, Melbourne, Victoria, Australia*

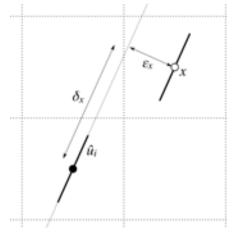


Figure 1: illustration of the distances used to set the neighbourhood weights (see Methods).

<u>Introduction</u>: Spherical deconvolution methods have been proposed to overcome the limitations inherent in the diffusion tensor model in regions of crossing fibres[1,2]. While spherical deconvolution is very sensitive to noise, the problem is greatly reduced by introducing a non-negativity constraint[3,4]. These improved methods are already being routinely used to obtain high quality results. However, there are two types of data where these methods provide limited results: (i) clinical 'DTI' data, acquired at relatively low *b*-values of ~1000s/mm^2 with relatively few DW directions (~12-20); these types of historical data are commonplace, and difficult to analyse robustly using HARDI methods in general due to their coarse angular sampling and relatively low overall contrast-to-noise ratio (CNR). (ii) high *b*-value, high spatial resolution HARDI data, where reduced signal-to-noise ratio (SNR) also leads to poor overall CNR, as well as significant problems with Rician bias. The aim of this study is to incorporate Rician bias correction and a constraint to enforce smoothness along fibre directions into the non-negativity constrained spherical deconvolution framework to allow the robust processing of such datasets.

<u>Methods</u>: We extend the constrained spherical deconvolution (CSD) approach of Tournier et al.[3], which we briefly summarise here. Starting from a heavily filtered initial estimate of the fibre orientation distribution (FOD) (obtained using an $l_{\max}$=2 linear spherical deconvolution), the amplitude of the current FOD estimate f_n is evaluated along a set of 300 uniformly sampled directions, and any directions with negative amplitudes are used to form the Tikhonov constraint matrix N_n that evaluates the FOD amplitudes along those directions only. This updated problem is then solved using ordinary least-squares to obtain the updated FOD estimate f_{n+1}:

$$f_{n+1} = \underset{f}{\operatorname{argmin}}\left[\|Mf - d\|_2^2 + \lambda_{\text{neg}}\|N_n f\|_2^2\right] \quad (1)$$

where M is the spherical convolution matrix relating FOD coefficients to DW signal intensities, d is the vector of measured DW signals, and λ_{neg} is the weight of the non-negativity constraint. A new constraint matrix N_{n+1} is then formed from f_{n+1} for use in the next iteration.

In this work, we also include a Rician bias correction approach, using an approximate equation for the expectation of the measured signal $s = R(a, \sigma) = \sqrt{(a^2 + 1.12\sigma^2)}$, given actual signal a and noise σ (similar to that proposed in [5]). The first term in eqn. 1 is modified to be the sum of squares between the measured signals and their predicted values $R(a, \sigma)$. This can be expressed as a least-squares problem by applying the opposite correction to the measured signals d, i.e. $d_n = d + a_n - R(a_n, \sigma)$, with $a_n = M f_n$ (note that we have extended the definition of $R(a, \sigma)$ to apply independently to each element of the vector a).

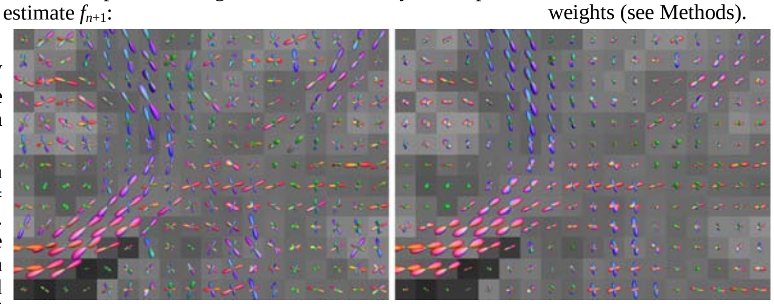


Figure 2: coronal projection of FODs in the centrum semiovale, obtained from a *b*=1000s/mm^2, 2 mm isotropic voxel size, 12 DW directions data set, processed using standard CSD (left) and including Rician correction & neighbourhood regularisation (right).

We also include a smoothness constraint along fibre orientations, based on the anisotropic weighted average of the current FOD estimates in the neighbourhood g_n:

$$g_n = \textstyle\sum_{x \in V} Q_x f_n(x) \quad (2)$$

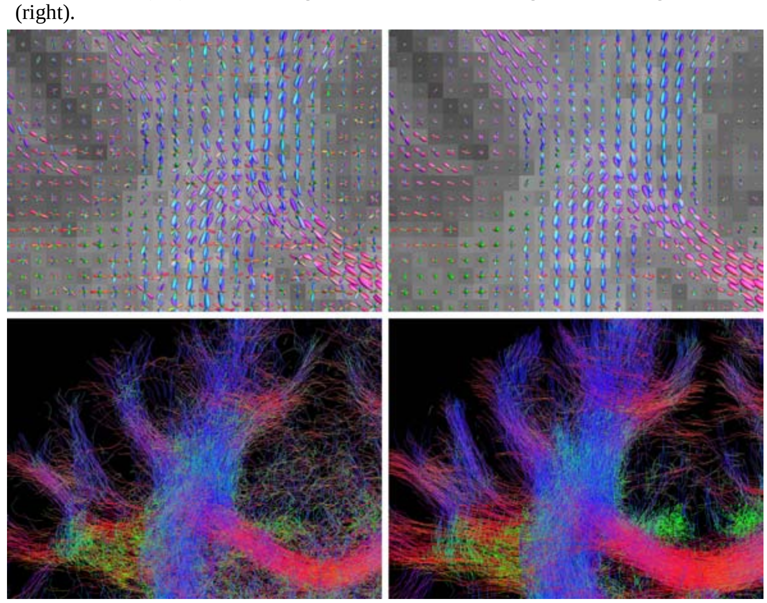
where x refers to a voxel in the neighbourhood V of the current voxel of interest, and Q_x applies directional weighting to its input FOD, to ensure smoothing is only performed over similar directions. The latter is computed as:

$$Q_x = N^{-1} W_x N, \qquad \text{with } W_{x;i,i} = \exp\left(-\frac{\delta_x^2}{2\sigma_\delta^2} - \frac{\varepsilon_x^2}{2\sigma_\epsilon^2}\right) \quad (3)$$

where δ_x is the projected distance to voxel x along direction vector $\hat{u}_i$, ε_x is the corresponding perpendicular distance (see Fig. 1), and W_x is a diagonal matrix. Briefly, this projects the FOD onto amplitudes along the set of (300) sampled directions, weights each contribution according the anisotropic weights W_x, and converts back to FOD coefficients. The standard deviation terms control the size of the neighbourhood; in this study, values used were σ_δ = 1 voxel and σ_ε = 0.425 voxel (i.e. FWHM=1 voxel). This is then included as an additional Tikhonov constraint with weight λ_{smooth}, to give the following equation for the full problem:

$$f_{n+1} = \underset{f}{\operatorname{argmin}}\left[\|Mf - d_n\|_2^2 + \lambda_{\text{neg}}\|N_n f\|_2^2 + \lambda_{\text{smooth}}\|f - g_n\|_2^2\right] \quad (4)$$

which can readily be solved using standard least-squares methods.



<u>Results</u>: The proposed approach was assessed on two datasets: a 'DTI' dataset (Siemens 3T TIM Trio, 12 channel head coil, 12 DW directions, *b*=1000s/mm^2, 2mm isotropic voxel size, 112×112 matrix, 70 slices); and a high-resolution dataset (Siemens 3T Skyra, 32 channel head coil, 64 DW directions, *b*=3000s/mm^2, 1.2mm isotropic voxel size, 192×192 matrix, 100 slices). Results are shown in figures 2 & 3 respectively. In all cases, convergence was achieved within 10 iterations, leading to reconstructions times of ~9min (for the 'DTI' dataset) and ~30min (for the high-resolution dataset) on a standard workstation computer.

<u>Discussion and Conclusion</u>: the proposed approach resulted in significantly improved results in both datasets , particularly for the high resolution dataset, at the expense of a loss of angular resolution compared to the original CSD. This suggests that the Rician bias correction and neighbourhood weighting are particularly beneficial for these types of data. However, for the high resolution dataset the SNR in the centre of the brain was too low for the approach to provide useable results, although improvements were nonetheless evident (data not shown). Based on this dataset, the minimum SNR where this approach can provide reasonable results is approximately 5, while good results were obtained in the periphery where the SNR was approximately 8. This approach should allow higher resolution data to be processed more reliably than is currently feasible, and can be used to improve results for dataset originally intended for DTI analyses, provided they were acquired using at least ~12 DW directions.

<u>References</u>: [1]Tournier et al., NeuroImage 23:1176–1185 (2004). [2]Anderson, MRM 54:1194–1206 (2005). [3]Tournier et al., NeuroImage 35:1459–1472 (2007). [4]Dell'Acqua et al., IEEE Trans Biomed Eng 54:462–472 (2007). [5]Gudbjartsson & Patz, MRM 34:910–914 (1995).