Estimation of White Matter Fiber Orientations with the Funk-Radon and Cosine Transform

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Introduction

MR tractography methods depend on the accurate estimation of orientation distribution functions (ODFs) from diffusion MRI data. A new ODF estimation method called the Funk-Radon and Cosine Transform (FRACT) was recently introduced for this purpose [1]. The FRACT is a linear method that generalizes the previous Funk-Radon Transform (FRT) [2]. While the FRACT is as easy to characterize as the FRT, it offers substantially improved angular resolution because it is able to approximate the constant solid angle ODF. This work evaluates the FRACT with respect to other ODF estimation techniques, including the FRT, constant solid angle q-ball imaging (CSA-QBI) [3], and non-negativity constrained super-resolution spherical deconvolution (CSD) [4].

Theory

The FRT computes an ODF as shown in (1), where $E(\mathbf{q})$ is diffusion data measured on a sphere in q-space, and $\delta(\cdot)$ is the Dirac delta function. The advantage of the FRT is that it can be computed easily using a spherical harmonic representation [5], and can be characterized theoretically. In particular, it can be shown that the FRT provides an approximation of the ODF definition given in (2), where $f(\cdot)$ is the ensemble average diffusion propagator that summarizes the diffusion characteristics within a given voxel. The peaks of this ODF can be used to infer the orientations of fibrous material in the body (e.g., brain white matter), since diffusion is frequently less-restricted along these orientations. The FRACT extends the FRT by computing (3), where the transform kernel is defined in (4). The parameter ξ can be adjusted to change the characteristics of the FRACT. The derivations in [1] showed that the FRACT approximates the improved constant solid angle ODF definition [1,3] given in (5). The transform kernels $\delta(\mathbf{u}^T\mathbf{q})$ and $G(\mathbf{u}^T\mathbf{q})$ for the FRT and the FRACT are plotted on the sphere in Fig. 1.

$$(1)\quad ODF_{FRT}(\mathbf{u}) = \int_{\mathbf{q}\in\mathbb{S}^2} E(\mathbf{q})\delta(\mathbf{u}^T\mathbf{q})d\mathbf{q}$$

$$(2)\quad ODF_{FRT}(\mathbf{u}) \approx \int_0^\infty f(\alpha\mathbf{u})d\alpha$$

$$(3)\quad ODF_{FRACT}(\mathbf{u}) = \int_{\mathbf{q}\in\mathbb{S}^2} E(\mathbf{q})G(\mathbf{u}^T\mathbf{q})d\mathbf{q}$$

$$(4)\quad G(t) = 2\,\delta(t) - \delta(t-\xi) - \delta(t+\xi)$$

$$(5)\quad ODF_{CSA}(\mathbf{u}) = \int_0^\infty f(\alpha\mathbf{u})\alpha^2 d\alpha$$

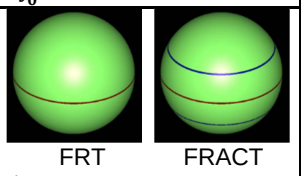

Fig. 1. Transform kernels for the FRT and the FRACT.

Materials and Methods

In vivo diffusion MRI data from the brain of a healthy adult was acquired on a 3T scanner (2x2x2 mm^3 resolution, 144 gradient directions, b-value=2500 s/mm^2). High anisotropy voxels in this dataset were isolated and assumed to represent a single fiber orientation. Data from multiple single-orientation voxels was combined to create a numerical crossing fiber phantom. The FRACT (spherical harmonic order = 8, $\xi = 0.34$), FRT (spherical harmonic order = 8), CSA-QBI (spherical harmonic order = 8), and CSD (spherical harmonic order = 12) were all applied to this data set. In addition, whole brain tractography was performed using DTK (http://trackvis.org) and visualized using the new diffusion tools available in the BrainSuite software package [6].

Results and Discussion

Results of applying the various methods to the crossing fiber phantom are shown in Fig. 2, with the true orientations shown with colored lines. The FRACT, CSA-QBI, and CSD methods all have substantially higher angular resolution than the FRT, and enable accurate identification of the fiber orientations. Both the FRACT and CSA-QBI have similar accuracy, though the FRACT has sharper ODF peaks in this case. CSD has very sharp ODF peaks, but larger angular error compared to CSA-QBI and the FRACT. This can be attributed to the fact that the CSD method is sensitive to the choice of the fiber response function that is deconvolved from the diffusion data. In the presence of heterogeneous white matter fibers, it is not possible to choose a single fiber response function that models all of the fibers well. CSD results using different fiber response functions had similar types of inaccuracies. Both CSA-QBI and CSD are nonlinear and make modeling assumptions that are frequently not satisfied in real data; on the other hand, the FRT and FRACT are linear and model-free, meaning that their performance is easier to predict. Brain tractography results are shown in Fig. 3, illustrating the improved tracking that is enabled by the FRACT.

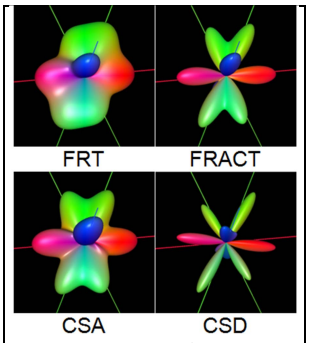

Fig. 2. Comparison of the FRACT, FRT, CSA-QBI, and CSD on the numerical crossing fiber phantom.

Conclusion

This work evaluated the FRACT relative to several other state-of-the-art ODF estimation methods that have been proposed in the literature. The FRACT was demonstrated to yield good performance relative to these other techniques, supporting the idea that it can be a powerful tool for improving MR tractography.

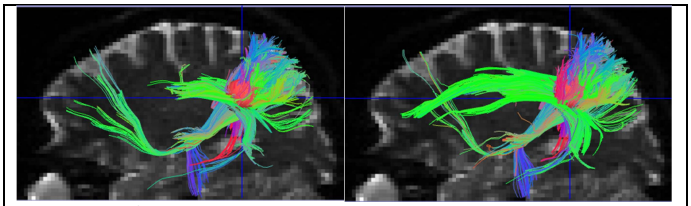

Fig. 3. FRT (left) and FRACT (right) tractography results.

References

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