SIGNAL AND NOISE PROPAGATION IN MR RECEIVE ARRAYS: THEORY AND EXPERIMENTAL VALIDATION

Matteo Pavan[1], David O. Brunner[1], Manuel Schneider[1], and Klaas Paul Pruessmann[1]
[1]*ETH and University Zurich, Zurich, Switzerland*

Introduction: In human MRI predominantly receive coil arrays are used due to their higher SNR gain, increased coverage and parallel imaging capability. SNR is maximized when the array has high filling factor, is built using low loss components, its Low Noise Amplifiers (LNAs) are properly matched and, as generally believed, its coil elements are decoupled in order to reduce noise correlation[1].

However, some authors[2] speculate that correlation of the noise resulting from channel coupling might not be as injurious as believed because it can be taken into account in the reconstruction and recent work[3] reveals an experimental case in which high noise correlation could be tolerated in order to increase the SNR performance of a two channel array. A theoretical approach that could explain such phenomena have been expound[4], it is formulated by means of the impedance matrix formalism but it lacks discrimination between effects of the matching network and effects of the array itself on the delivered SNR.

This work aims to address a new theoretical understanding of SNR behavior depending on array's matching condition expressed in the scattering matrix formalism with clear distinction between the impact of different components along the receive chain. An experimental validation is also given.

Theory: The multichannel receiver system can be regarded as a cascaded series of linear multiport as shown in Fig. 1. Signal propagation in electrical devices is characterized at typical MRI frequency by means of the scattering matrix; such formalism can be extended by means of noise waves[5] in order to characterize noise performance as well. A noise wave is a random complex vector (indicated in figure 1 by the letter **n**) which sums up with the outgoing **b** wave of the scattering matrix formalism; its statististical behavior in conjunction with all other ports of the device is described by means of a covariance matrix $\boldsymbol{\Psi}=\mathbf{E}[\mathbf{n}\mathbf{n}^{\mathbf{H}}]$ (**E** indicates the expectation). For a passive device like a Matching Network (MN) or a coil, **Ψ** is given by Bosma theorem[6] and for an active one like an amplifier it can be written[5] based on its noise parameters F_{MIN}, Z_{OPT} and R_n.

Given a source multiport generating a signal vector $\mathbf{b_S}$, one can calculate its correspondent output wave $\mathbf{b_L}$ as seen at the load using elementary RF circuit theory which implies the use of only the **S** matrix of each multiports present in the cascade; since noise-waves propagate and are scattered in the same way as the signal-wave $\mathbf{b_s}$ does, it is straightforward to calculate[6] the total noise covariance matrix $\boldsymbol{\Psi}_{\mathbf{TOT}}$ seen at the load as a sum of the individual covariance matrices $\boldsymbol{\Psi}_{\mathbf{S}}$, $\boldsymbol{\Psi}_{\mathbf{MN}}$, $\boldsymbol{\Psi}_{\mathbf{LNA}}$ and $\boldsymbol{\Psi}_{\mathbf{L}}$, each weighted by a noise transformation matrix[6] **N** (that depends only on **S** matrices) which represents the total gain or attenuation (including mismatch) of the multiports in which the noise wave has to propagate through: $\boldsymbol{\Psi}_{\mathbf{TOT}} = \mathbf{N_1}\boldsymbol{\Psi}_{\mathbf{S}}\mathbf{N_1}^{\mathbf{H}} + \mathbf{N_2}\boldsymbol{\Psi}_{\mathbf{MN}}\mathbf{N_2}^{\mathbf{H}} + \mathbf{N_3}\boldsymbol{\Psi}_{\mathbf{LNA}}\mathbf{N_3}^{\mathbf{H}} + \mathbf{N_4}\boldsymbol{\Psi}_{\mathbf{L}}\mathbf{N_4}^{\mathbf{H}}$.

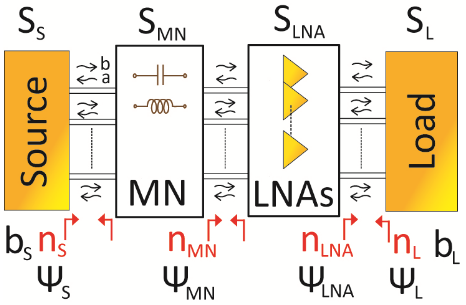

Figure 1: General noise wave model; **S** is the scattering matrix, **n** a noise vector, **Ψ** its covariance matrix and $\mathbf{b_S}$ the signal generated by the source multiport.

Materials and Method: A 2 channels receive coil array was tuned and matched (see figure 2), one channel was connected directly to a high impedance LNA and the second channel was connected to a LNA by means of an Automatic Matching Network (AMN)[3, 7] which consists of PI matching network containing three varicaps. 50 dynamic were measured when changing impedance matching of the AMN before the beginning of each experiment, impedance was changed by progressively increasing varicaps voltage. SNR maps were acquired (see Fig. 3) by means of a 3T MR system (Philips Achieva, Philips Healthcare, Best, NL) using as phantom a 10cm diameter water bottle. SNR maps were acquired by a gradient echo sequence (TE=7.1msec, TR=16msec, voxel=0.47x0.47x10mm^3, flip angle=10°, 3 averages, 256X256 pixels).

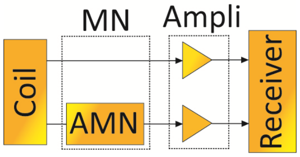

Figure 2: Experimental setup

The signal vector at the coil was calculated and then used to predict output signal vector and total noise covariance matrix for all 50 matching situations, eventually the SNR was calculated by means of $\mathrm{SNR}_{\mathrm{Roemer}}{}^2 = \mathbf{b_L}^{\mathbf{H}}\ (\boldsymbol{\Psi}_{\mathbf{TOT}})^{-1}\ \mathbf{b_L}$; Basic RF circuit theory was used to transform waves into voltages in order to predict the noise correlation coefficient too. SNR and noise correlation coefficient from measured MR data was then compared with the predicted one in a single pixel basis. The experiment was performed once for a coupled loop pair and once for a geometrically decoupled situation.

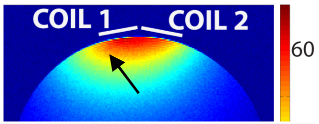

Figure 3: measure SNR with indicated pixel under investigation

Result: Single pixel SNR and noise correlation coefficient predicted using the theory are generally in good agreement with the measured SNR and measured noise correlation coefficient both for the case when the coils are geometrically decoupled and also for the one when the coils are coupling (see Fig. 4), however there are at least two outlier points where the SNR prediction is too high and the predicted noise correlation coefficient in the case of a decoupled coil (Fig. 4, top right) is generally underestimated.

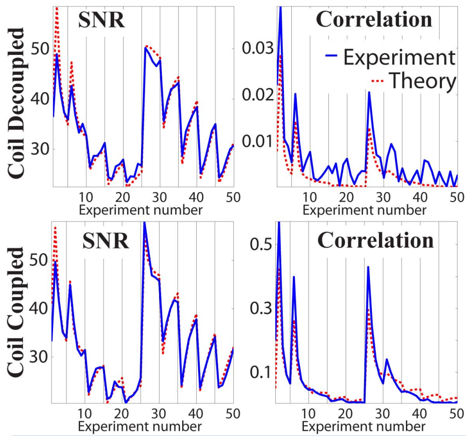

Figure 4: Measurement-Theory comparison (single pixel)

Discussion and Conclusion: A new general theory for prediction of SNR and noise correlation in MRI receive-coil arrays has been developed; in contrast to previous work[4] it distinguishes noise produced from different sources such as coil, matching and amplification stage. A comparison of the predicted SNR with the measured one shows in general good agreement but noise correlation coefficient in the case of a decoupled coil is typically underestimated; a probable reason for this might be the fact, that contributions to the signal and the noise behavior from common mode currents are generally neglected.

A good SNR prediction relies on the accuracy of the scattering matrices and LNAs noise parameters measurement as performed by the network analyzer. Since the theoretical calculation involves matrix inversions of said scattering matrices, ill-conditionings can amplify measurement errors disproportionally.

As seen in previous work[3] SNR was found to be high for matching conditions where the total noise correlation was high too, the reason for this is that SNR calculation takes correlation into consideration.

The presented theory delivers a detailed understanding of the SNR yield of multi-channel receiver systems and can hence be generally used for the design of all involved components and their interplay. ***References:*** 1. Roemer P.B. et al. MRM 16:192-225 (1990) | 2. Ohliger MA et al. MRM 52: 628-639 (2004) | 3. Pavan M. et al. Proc. ISMRM 2012 | 4. Findeklee C. IEEE Trans. AP 59: 452-459 (2011) | 5. Scott W.W et al. IEEE Trans. MTT 40:2004-2012 (1992) | 6. Bosma H. PhD thesis, Technical University Eindhoven (1967) | 7. Biber S. et al. Proc. ISMRM 2009