Genuine minimum power gradient coil design accounting for gaps between tracks or wires

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Target audience: This work will be of interest and benefit primarily to system designers and hardware engineers, in particular those working on gradient coils.

Purpose: Gradient coil design is an optimization problem subject to a variety of constraints depending on the specific application. For small coils, such as gradient inserts or those used in small animal systems or in MR microscopy, the primary design criterion is typically the minimization of dissipated power. It is common to solve the optimization problem in terms of a distributed current density on the coil surface and to minimize dissipated power whilst maintaining an acceptable level of gradient field homogeneity. This current density is then approximated using a finite number of either coil tracks of variable width (for the case of water-jet cutting or chemical etching a copper sheet) or wires of fixed width (for the case of winding by hand/machine for simplicity). While this approximation is sufficient in terms of field accuracy and inductance calculations, the existence of a finite gap width between tracks/wires can lead to gross errors in the calculation of coil resistance, resulting in a coil that is no longer optimal in terms of power dissipation.[1,2] The purpose of the presented work is to provide a current density mapping that accounts for this approximate discretization and yields truly optimal minimum power coils for any gap width or chosen method of manufacture.

Methods: We consider firstly the general case of approximating a current density, $\boldsymbol{j_c}$ (A/m), with tracks of variable width. The centre line of each track is usually obtained by taking equally spaced contours of the associated streamfunction, such that each track carries a fixed current, I (A). Furthermore, there exists an absolute minimum gap width, g, between adjacent tracks and also a fixed maximum track width, w, such that the current does not deviate far from its intended path. In regions of high continuous current density, in which the minimum gap width becomes an appreciable fraction of the streamfunction contour separation, s, the real current density within the tracks will therefore be much higher than $\boldsymbol{j_c}$ (indeed $\to\infty$ as $s\to g$). Similarly, in regions of low continuous current density, in which $s \geq w+g$, the real current density within the tracks will be fixed at I/w and zero elsewhere. Therefore, accounting additionally for the appropriate contribution to the total power, we propose the following mapping to the "real" current density $\boldsymbol{j_t}$:

$$|j_t| = \begin{cases} \sqrt{\frac{I}{w}|j_c|}\,, & \text{if} \quad |j_c| \leq \frac{I}{w+g} \\ \dfrac{|j_c|}{\sqrt{1-\frac{g}{I}|j_c|}}\,, & \text{if} \quad \frac{I}{w+g} < |j_c| < \frac{I}{g} \\ \infty\,, & \text{if} \quad |j_c| \geq \frac{I}{g} \end{cases}$$

Minimizing the total power associated with the new mapped current density $\boldsymbol{j_t}$ will now result in the optimum coil post-discretization. In the present work, both x- and z-gradient coils were considered with geometry appropriate to small animal imaging systems (or gradient inserts) of various lengths (radius 0.11 m). The current density was represented using Fourier series with 50 modes. The optimization was performed using the function *fmincon* from MATLAB's Optimisation Toolbox (interior point method). The starting guess for this iterative method was a standard minimum power coil with 5% maximum field error within a 0.12 m DSV. The norm of the field error over the DSV was fixed throughout to maintain field accuracy. The method was also extended in a straightforward manner to the fixed wire width case by imposing an additional constraint on the minimum separation of streamfunction contours, such that $\min(s) \geq w+g$. The program FastHenry2 was used to simulate coil resistance, R, and results were compared using the common figure of merit, η^2/R, which is inversely proportional to dissipated power (η is coil efficiency).

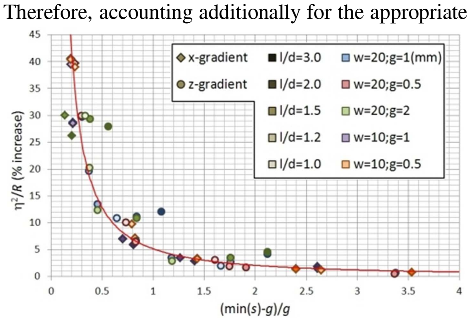

Fig.1: % increase η^2/R for variable track opt.

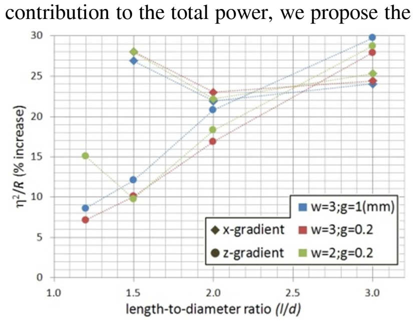

Fig.2: % increase η^2/R for fixed wire opt.

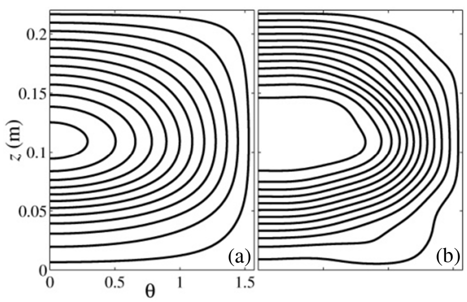

Fig.3: (a) Orig. minP, (b) fixed wire opt.(w=3;g=1)

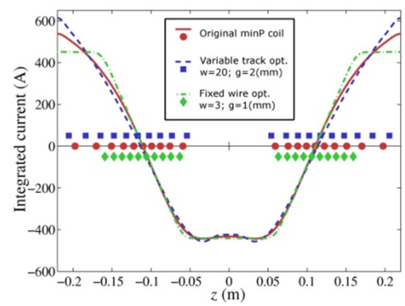

Fig.4: z-grad. streamfn. & winding pos.

Results: A survey of a great number of different cases was performed, for both x- and z-gradient coils, with either variable track or fixed wire width coil windings. The impact was tested of varying the length of the coil (length-to-diameter l/d ratio), the number of coil windings, the minimum gap width, and the maximum track width or wire width. Fig. 1 displays the improvement in η^2/R for all the variable track cases considered, plotted against $(\min(s)-g)/g$ of the corresponding original minimum power coil, which was found to be the primary indicator for expected improvement. Fig. 2 shows the improvement in η^2/R plotted against l/d for the fixed wire width cases. Some example coil winding results are displayed in Figs. 3-4 for x-gradients and z-gradients, respectively, to demonstrate the change in winding position.

Discussion: For the variable track cases (both x- and z-gradients), the improvement in η^2/R was found to be appreciable only for coils in which the minimum separation of the original minimum power coil was less than twice the gap size (Fig. 1). At this point, one may expect ~5% improvement in η^2/R by performing the power minimization on the mapped current density. If the ratio of $\min(s)/g$ is lower at 1.6, one might expect ~10% improvement, and so on. As such, for variable track coils, the method becomes useful only for densely packed windings, and otherwise the original minimum power coil is a good approximation. However, for the fixed wire width cases, for which the discretization represents a more extreme approximation to the continuous current density, the benefit of using the proposed mapping is substantial (Fig. 2). For all the x-gradient coils considered and all the longer z-gradient coils, improvements in η^2/R of 15-30% were obtained consistently. Interestingly, the windings for these coils are approximately centred on the traditional Golay and anti-Helmholtz arrangements, for the x- and z-gradients, respectively (see Figs. 3-4).

Conclusion: A method has been presented for designing genuine minimum power gradient coils, accounting for the approximation during discretization due to minimum gap width and maximum track/wire width (i.e. for when the build method is known prior to coil design). For the case of coils constructed using copper tracks of variable width, the results suggest that it is beneficial (>5% improvement) to use the presented method over standard approaches when the minimum separation between streamfunction contours is less than twice the minimum gap width. However, the greatest utility of the method is for designing coils to be constructed using wire of fixed width. For these coil types the method provides very substantial improvements in all cases and alters the winding structure considerably from the standard minimum power designs. These new genuine minimum power designs would possess significantly lower average temperatures and improved duty cycle limits.

References: [1] Chu KC, Rutt BK. MR gradient coil heat dissipation. *Magn. Reson. Med.* 1995; 34:125-132. [2] Poole MS, While PT, Sanchez Lopez H, Crozier S. Minimax current density gradient coils: analysis of coil performance and heating. *Magn. Reson. Med.* 2012; 68:639-648.