Linear Phase Shift Correction for Field Map Estimation with Bipolar Gradient Dual-Echo Sequence using the Noise PDF

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INTRODUCTION: Dual echo fast field echo (DEFFE) sequence with two back-to-back readout gradients in opposite polarity acquires two echoes efficiently with one RF pulse at a desired short $\Delta T_{\mathrm{E}}\ (= TE2 - TE1)$ without inter-echo alignment errors for field map estimation. Oftentimes, a static field map is estimated by taking the phase difference of a pair of field echo, i.e., gradient echo, images acquired separately at two different echo times [1,2]. While a static field map from a phantom can be accurately estimated, the separate single volume acquisition approach is prone to motion-induced and position-dependent errors in human subjects. Changes in B_0 during the time delay between the separate volume acquisitions, typically 3-4 minutes per acquisition, due to the head displacement or physiological brain motion, cause measurement errors and degradation of field maps [3]. Dual-echo images may be acquired by using the same positive polarity in the readout gradient, however, ΔT_Emay be lengthened, thus more prone to the phase wrap error. Previously, the phase shift error (α) from the sequence specific, asymmetric bipolar readout pulses were modeled as an affine term in the readout direction using motionless phantom data, as the first and second echoes are misplaced by a temporal delay of the readout gradient field, as illustrated in [3]. In this work, we introduce an appropriate probability density function (PDF) of non-uniform noise in the phase difference images. The estimator derived from the noise PDF is advantageous in finding α in that (i) it is not affected by the wrapping effect and (ii) the outliers in the given phase difference images are effectively removed by the weights given by the PDF, where the weight is a function of signal intensity, I.

METHODS: A field map, $\Delta\omega$, is acquired from the phase difference map θ of the two complex-valued images $I_{\mathrm{TE1}}^{\mathrm{single}}$ and $I_{\mathrm{TE2}}^{\mathrm{single}}$acquired from a phantom using a single-echo sequence separately with ΔT_{E} as $\Delta\omega = \theta/\Delta T_{\mathrm{E}}$, whereas the phase difference map ψ_{dual} from dual-echo images carries sequence-specified errors other than θ due to the echo delay. Letting $I_{\mathrm{TE1}}^{\mathrm{dual}}(\mathbf{r})$ and $I_{\mathrm{TE2}}^{\mathrm{dual}}(\mathbf{r})$ be the dual-echo complex-valued images, where $\mathbf{r} = [x, y, z, q]^T$ represents the voxel coordinate (x, y, z) and the coil index (q) of the multi-coil scanner, we denote $\psi_{\mathrm{dual}} = \angle\left(\bar{I}_{\mathrm{TE1}}^{\mathrm{dual}} \cdot I_{\mathrm{TE2}}^{\mathrm{dual}}\right) \in [-\pi, \pi]$, where the bar (e.g. $\bar{I}$) represents the conjugate. With the linear phase shift αx, $\psi_{\mathrm{dual}} = \theta + \alpha x + \beta$, where β represents a residual object specific global shift [3]. Empirically we define,

$$\phi(\mathbf{r}) \triangleq \psi_{\mathrm{dual}}(\mathbf{r}) - \theta(\mathbf{r}) = \alpha x + \beta \pm 2n\pi + \varepsilon(\mathbf{r}) \quad \in [-\pi, \pi] \tag{1}$$

where ε is zero-mean additive (real-valued) noise and $\pm 2n\pi$ represents the wrapping effect with some integer number n. Assuming that the noise in the image pairs, $I_{\mathrm{TE1}}^{\mathrm{single}}$ and $I_{\mathrm{TE2}}^{\mathrm{single}}$, and $I_{\mathrm{TE1}}^{\mathrm{dual}}$ and $I_{\mathrm{TE2}}^{\mathrm{dual}}$, are all *i.i.d.* zero-mean complex Gaussian with variance of $\sigma_{\mathrm{single}}^2$ and σ_{dual}^2, respectively, we approximate the PDF of the noise ε in (1) as

$$p(\varepsilon(\mathbf{r})) \sim \exp\{\kappa(\mathbf{r}) \cdot \cos\varepsilon(\mathbf{r})\} \tag{2}$$

where

$$\kappa^{-1}(\mathbf{r}) = \frac{\left(\left|I_{\mathrm{TE1}}^{\mathrm{single}}(\mathbf{r})\right|^2 + \left|I_{\mathrm{TE2}}^{\mathrm{single}}(\mathbf{r})\right|^2\right)\sigma_{\mathrm{single}}^2 + \sigma_{\mathrm{single}}^4}{\left|I_{\mathrm{TE1}}^{\mathrm{single}}(\mathbf{r})\right|^2\left|I_{\mathrm{TE1}}^{\mathrm{single}}(\mathbf{r})\right|^2} + \frac{\left(\left|I_{\mathrm{TE1}}^{\mathrm{dual}}(\mathbf{r})\right|^2 + \left|I_{\mathrm{TE2}}^{\mathrm{dual}}(\mathbf{r})\right|^2\right)\sigma_{\mathrm{dual}}^2 + \sigma_{\mathrm{dual}}^4}{\left|I_{\mathrm{TE1}}^{\mathrm{dual}}(\mathbf{r})\right|^2\left|I_{\mathrm{TE1}}^{\mathrm{dual}}(\mathbf{r})\right|^2}. \tag{3}$$

The advantage of the PDF is that it is differentiable and tolerant of the wrapping effect (since $\cos(\phi \pm 2n\pi) = \cos(\phi)$). Having introduced the data model Eq (**1**) and the noise PDF with weighting, Eq (2), we find the parameters α and β by the steepest descent method using the maximum likelihood estimator,

$$\max_{\alpha,\beta} \sum_{\text{for all } \mathbf{r}} \kappa(\mathbf{r}) \cdot \cos(\phi(\mathbf{r}) - \alpha x - \beta) \tag{4}$$

MRI simulation and data acquisition: We conducted the linear phase correction using simulated and acquired phantom and human data. We simulated a phase difference map with moderately strong wrapping ($\pm 2\pi$) effects by setting the parameter α beyond its probable extent to demonstrate the effectiveness of our method. As we determined the parameter α to be typically ~2.0 for our DEFFE sequence, we set $\alpha = 6.60$ for the simulation. We acquired MRI data from a gel filled sphere phantom (dia=20cm) and a human subject in Philips 3T (Best, Netherland) Ingenia system. Two separate volumes were acquired using a fast field echo sequence with $T_R/T_E = 12/4.1$ and $12/5.1$ ($\Delta T_E = 1.0$), image resolution $1 \times 1 \times 2$ (mm^3). Dual echo data were acquired using the DEFFE sequence with ($T_R/T_E/\Delta T_E = 703/4.1/1.0$), resolution $1 \times 1 \times 1$ (mm^3).

RESULTS: Fig 1 displays transverse magnitude images and phase difference maps before and after the phase corrections from dual echo data (normalized [0,1]). Fig 1 (a) shows the simulated phase difference map generated with $\alpha = 6.60$, $\beta = 0.10$, $\sigma_{\mathrm{single}}^2 = 6.0 \times 10^{-6}$, $\sigma_{\mathrm{dual}}^2 = 6.0 \times 10^{-5}$, and the corrected map with the estimated $\hat{\alpha} = 6.59$ and $\hat{\beta} = 0.10$ by Eq (4). Fig 1(b) shows result from the homogeneous phantom with the estimated parameters $\hat{\alpha} = 2.32$ and $\hat{\beta} = 0.19$. The noise variance of the acquired homogeneous phantom were estimated as $\sigma_{\mathrm{sigle}}^2 = 6 \times 10^{-6}$ and $\sigma_{\mathrm{dual}}^2 = 6 \times 10^{-5}$. Using the estimated $\hat{\alpha} = 2.32$, the human dual echo data were corrected as shown in Fig 1(c). The cross sectional plot in Fig 1(b), bottom, shows that, after the linear correction, the phase difference map from DEFFE is in good agreement with θ which serves as the truth.

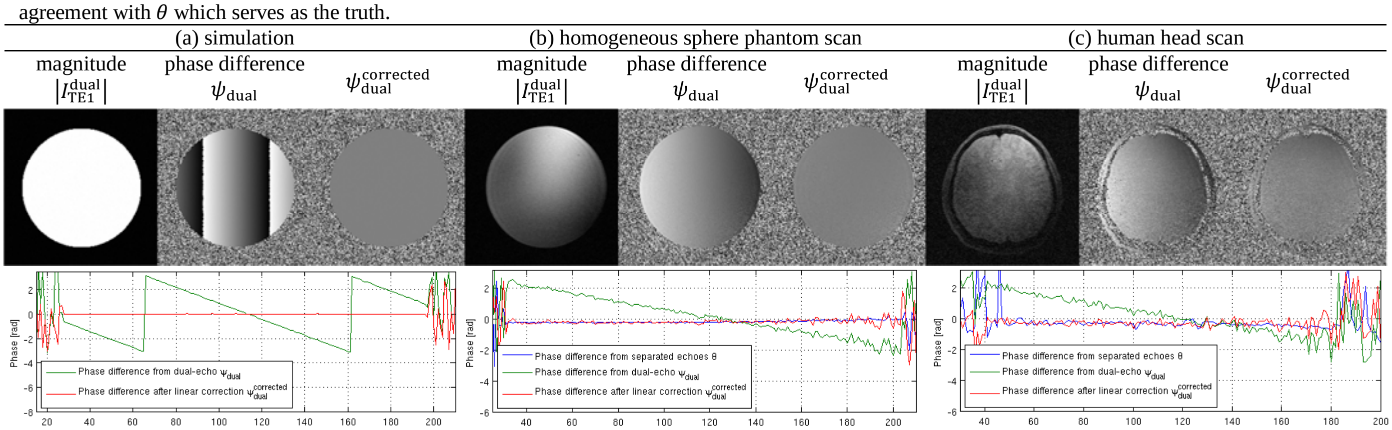

Fig 1: Linear phase correction for dual-echo data: (a) the simulation with severe wrapping effect, (b) a physical sphere phantom scan, (c) a human head scan. The parameter $\boldsymbol{\alpha}$ was estimated from the homogeneous phantom. The cross sections along x-axis of the phase difference images from the top row were plotted in the bottom.

CONCLUSION: We present the estimation of a residual phase error from the asymmetric readout pulses in a dual echo sequence used for field map estimation. Results from phantom and human data suggest that the linear phase error stays constant, hence, can be applied to different data acquired with the same protocol. The newly implemented noise PDF in Eq (2) proves to be effective for processing phase data for the field map estimation. Using an appropriate PDF, any pre-processing to remove the noisy phase data, i.e., segmentation or masking, may not be necessary as shown in Eq (3), nor phase unwrapping as demonstrated in the simulation.

REFERENCES:

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