Fast DSI Reconstruction with Trained Dictionaries

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TARGET AUDIENCE: Diffusion Weighted Imaging (DWI) investigators; neuroimaging scientists and clinicians.

PURPOSE: Significant benefit in Compressed Sensing (CS) reconstruction of Diffusion Spectrum Imaging (DSI) data from undersampled q-space was demonstrated when a dictionary trained for sparse representation was utilized [1] rather than wavelet and Total Variation (TV) [2]. However, computation times of both dictionary-based and Wavelet+TV methods are on the order of *days* for full-brain processing using standard workstation. This abstract presents two algorithms that are 3 orders of magnitude faster than these CS methods with reconstruction quality comparable to the previous dictionary-CS approach. The proposed methods reduce the in vivo reconstruction error up to 2 times compared to the Wavelet+TV algorithm with processing times on the order of *seconds per slice*. Comparison with respect to low-noise, 10-average fully sampled data reveals that the proposed algorithms at 3-fold acceleration give lower reconstruction error than the 1-average fully-sampled data, alluding to the denoising property of these algorithms which relies on prior information in the trained dictionary.

METHODS: Previously proposed methods [1,2] and two proposed methods are described below:

Wavelet+TV [2]: Letting $\boldsymbol{p} \in \mathbb{C}^N$ represent diffusion probability density function (pdf) at a particular voxel, and $\boldsymbol{q} \in \mathbb{C}^M$ denote the undersampled q-space data, the following is solved at a single voxel: $min_{\boldsymbol{p}} \|\mathbf{F}_\Omega \boldsymbol{p} - \boldsymbol{q}\|_2^2 + \alpha \cdot \|\boldsymbol{\Psi} \boldsymbol{p}\|_1 + \beta \cdot \mathrm{TV}(\boldsymbol{p})$, where $\mathbf{F}_\Omega$ is the undersampled Fourier transform, $\boldsymbol{\Psi}$ is a wavelet operator, $\mathrm{TV}(.)$ is the TV penalty, α and β are regularization parameters.

Dictionary-CS [1]: Given a training set of example pdfs, the K-SVD algorithm [3] is used to find a dictionary $\mathbf{D}$ that achieves maximal sparse representation of the training pdfs. For reconstruction, the FOCUSS algorithm [4] is used to solve: $min_{\boldsymbol{x}} \|\boldsymbol{x}\|_1 \;\; such\ that \;\; \mathbf{F}_\Omega \mathbf{D} \boldsymbol{x} = \boldsymbol{q}$, where $\boldsymbol{x}$ are the dictionary transform coefficients.

Proposed 1: Tikhonov regularization: Rather than seeking sparse solutions with respect to the dictionary $\mathbf{D}$, ℓ_2 regularization is employed by solving: $min_{\boldsymbol{x}} \|\mathbf{F}_\Omega \mathbf{D} \boldsymbol{x} - \boldsymbol{q}\|_2^2 + \lambda \cdot \|\boldsymbol{x}\|_2^2$, where λ is a regularization parameter.

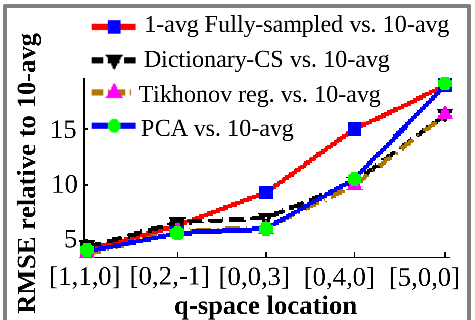


Fig.1 Error compared to 10-avg data collected at 5 q-space points

Proposed 2: Principal Component Analysis (PCA): After subtracting the average pdf $\boldsymbol{p}_{mean}$ from the training pdf dataset, PCA is applied to produce a matrix of principal pdfs $\mathbf{Q}$. A reduced-dimensionality representation is obtained by generating the matrix $\mathbf{Q}_T$ from the first T columns of $\mathbf{Q}$. The target pdf is estimated from undersampled q-space by solving: $min_{\boldsymbol{pca}} \|\mathbf{F}_\Omega \mathbf{Q}_T \boldsymbol{pca} - (\boldsymbol{q} - \mathbf{F}_\Omega \boldsymbol{p}_{mean})\|_2^2$, where $\boldsymbol{pca}$ are the PCA coefficients.

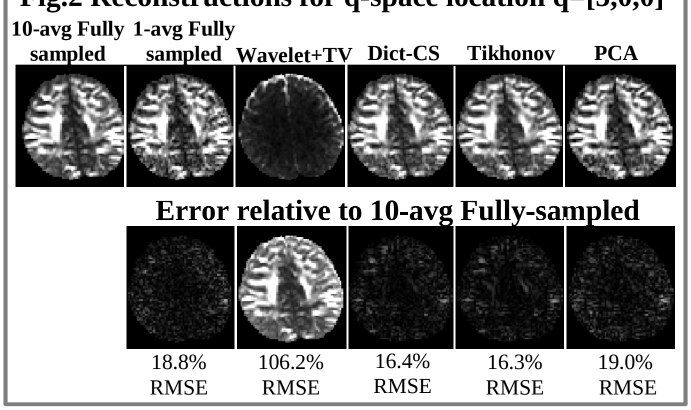
As the minimizers of the optimization problems in the proposed methods can be expressed in closed form, the computational cost is a single matrix-vector multiplication per voxel. Dictionary training for all methods is based on data from a subject *different* from the test subject. Optimal regularization parameters for all methods are determined using the training dataset, and are chosen to minimize the reconstruction error. 515 direction healthy volunteer DSI data with b_{max} = 8000 s/mm^2 at 2.3 mm isotropic resolution were acquired using a novel 3T system (MAGNETOM Skyra CONNECTOM, G_{max}=300 mT/m and Slew=200 T/m/s). To obtain low-noise ground truth, 10-avg data were also collected at 5 q-space points.

RESULTS: Fig.1 presents the q-space reconstruction error of the 5 q-space points, where low-noise ground truth is available, for the dictionary-based methods and 1-avg fully-sampled images. Fig.2 shows q-space reconstructions at the point on the outermost shell and differences to 10-avg data. Fig.3 depicts reconstruction errors in the pdf space at two different undersampling factors, R=3 and 9.

DISCUSSION: While the proposed methods have smaller error at R=9 compared to the Wavelet+TV results at the lower acceleration factor of R=3, they are also 1000-fold faster (Fig.3). Based on the low-noise 10-avg data comparison, the proposed methods at R=3 are seen to have slightly less error than the fully-sampled data (Fig.1). Fig.2 shows that the Wavelet+TV method substantially underestimates the high q-space content, while the proposed methods have error comparable to the fully-sampled data. Since the training data were obtained from a different subject, dictionary methods are seen to generalize across healthy subjects.

CONCLUSION: In place of typical ℓ_1-regularized algorithms, fast and simple ℓ_2-based algorithms were successfully employed to reconstruct undersampled data by relying on prior information extracted from a training dataset. The proposed methods demonstrate 1000-fold computational speed-up relative to the previous dictionary-based algorithm in [1] with comparable reconstruction quality.

Fig.2 Reconstructions for q-space location q=[5,0,0]



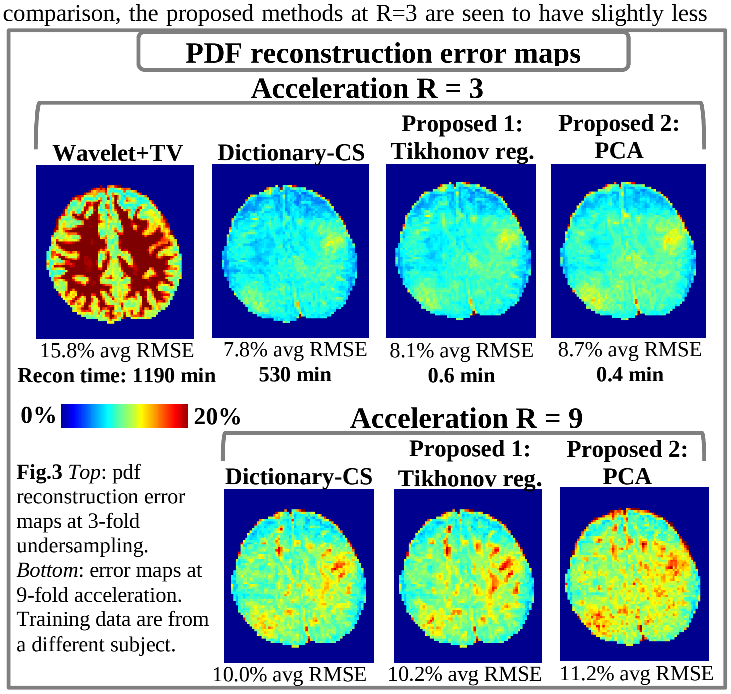


Fig.3 *Top*: pdf reconstruction error maps at 3-fold undersampling. *Bottom*: error maps at 9-fold acceleration. Training data are from a different subject.

REFERENCES: [1] Bilgic et al. MRM'12; [2] Menzel et al. MRM'11; [3] Aharon et al. IEEE TSP'06; [4] Gorodnitsky et al. IEEE TSP'97

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