# A novel undersampling scheme for data acquisition in non k-space domains

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## Introduction

By decreasing the acquisition time without loss of spatial resolution Compressed Sensing (CS) has revolutionized the field of Magnetic Resonance Imaging (MRI) [1, 2]. However, although images are reconstructed in a sparse transformation domain (e.g. Wavelet-domain), raw data is still acquired in the Fourier-domain (k-space), which provides only suboptimal sparsity. Novel developments in the field of multielement parallel transmit systems [3, 4] suggest that an arbitrary sample excitation is possible. Hence hypothetically image data can be directly acquired from a sparse image domain. In the present work a method for scan-time reduction using sparse-domain acquisition is introduced.

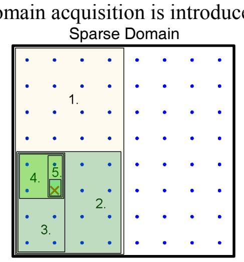


*Figure 1: Representation of a sparse domain containing all information in a single location (marked by a red cross). The successively scanned ranges are marked and labeled by scan numbers.*

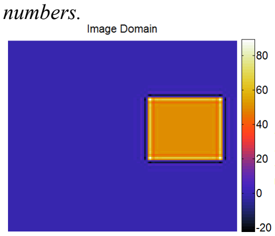


*Figure 2: Example excitation that can be used to scan the sum of multiple CDF 9/3 wavelets.*

| Stand. dev. | mean error | max error | min error |
|---|---|---|---|
| **10%** | 24.7% | 24.9% | 24.6% |
| **50%** | 13.5% | 14.0% | 12.8% |
| **90%** | 4.1% | 4.8% | 3.0% |

*Table 1: Error of $\|x\|_1$ from $\|y\|_1$ for a sparse phantom image with white noise of different intensity. The standard deviation is given in percent of the average pixel intensity.*

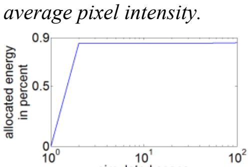


*Figure 3: Already captured image energy versus number of measurements in the presented scheme. The x-axis is scaled logarithmic.*

## Theory

In a sparse domain all image information is contained in a few coefficients. In order to reconstruct the image these coefficients need to be localized.

In the proposed algorithm, this is done by a binary search. To illustrate this idea, consider a space of one hundred binary coefficients $c_i \in \{0,1\}, 1 \leq i \leq 100$, with only one being non-zero $c_i = 1 \Leftrightarrow i = i^*$. The most efficient strategy to locate the non-zero coefficient is to determine whether it lies in $[1,50]$ or $[51,100]$. Depending on the result the next test should be whether it lies in $[1,25]$ or $[26,50]$ and so on. These can be tested by evaluating sums like $\sum_{i=1}^{50} c_i = 1 \Leftrightarrow i^* \in [1,50]$. Consequently the successively evaluation of sums over a decreasing range efficiently localizes the non-zero coefficients.

In the case of 2D MRI with acquisition in a sparse domain, $c_i$ are the coefficients in the transformation domain, e.g. wavelet coefficients. The resulting scanning scheme that is used to localize non-zero coefficients in the sparse domain is illustrated in Figure 1. As it is indicated there the next area to be scanned is determined online depending on the last measurement. An example excitation, which is necessary to scan the sum of multiple wavelets, can be seen in Figure 2.

Since it is not guaranteed that all coefficients are non-negative, decomposing a scanned area needn't necessarily be stopped if the last measurement equals zero. But if the energy that is saved in the sparse coefficients is known, scanning can be stopped if enough energy is captured. If $x \in \mathbb{C}^n$ describes the image and $y \in \mathbb{C}^n$ its representation in the sparsifying space, then $\|y\|_1 := \sum_i |y_i|$ describes the energy contained in the non-zero coefficients. Since this measure cannot be acquired directly, $\|x\|_1 := \sum_i |x_i|$ serves as a heuristic value. $\|x\|_1$ can be easily acquired, as a scan without encoding. To proof the accuracy of the heuristic value the two measures were compared for the sparse shepp-logan phantom with white noise of different intensities.

In the resulting scheme, the rectangles are further decomposed as long as their value exceeds a threshold. Since $\sum_i |y_i| \geq |\sum_i y_i|$, it is sure that the rectangle contains at least the measured energy. Only if all rectangles with enough energy are fully decomposed, the rectangle of the largest size, which is not further decomposed, is chosen to be scanned next.

## Materials

The method has been evaluated on image data of a healthy volunteer's head with the following parameters: 2D-FLASH, TE/TR=4ms/8.6ms, matrix-size=512x512, slice-thickness=7mm. Images were reconstructed and the image acquisition in the sparse image transformation was simulated. Various undersampling factors were achieved by using different thresholds. The results were compared to low resolution images, which can be acquired with the same number of measurements in the k-space. Both images were reconstructed on a 512x512 matrix to compare them with the high resolution image. The wavelet domain with CDF 3/9 wavelets was chosen as the transformation space. All evaluations were carried out in MATLAB® and the WAVELAB850 [5] toolbox was used for wavelet processing.

## Results and Discussion

As it can be extracted from Table 1, $\|x\|_1$ is at least a rough heuristic value. The results indicate that accuracy decreases with the sparsity of the image. In Figure 2 one can see the current energy versus the simulated measurement process of the phantom. It is obvious that the first scans allocate most of the image energy. Hence, the requirements on the accuracy of the heuristic are sufficient low.

In Figure 4 one can see a performance comparison of the herein presented scheme with regular k-space sampling. The upper row shows the results for k-space sampling, while the images with sparse-domain sampling are listed in the lower row. The images can be compared to the original on the very left. Using the novel scheme the normalized root mean square error (NRMSE) of the undersampled from the original image was 6.56% with 15% of the measurements, 11.2% with 8.2% and 12.9% for 3.5%. Compared to these values, the k-space sampling achieved 36.3%, 65.9% and 86.7% respectively. Therewith the improvement in the NRMSE was over 80%.

## Discussion

It can be seen in Figure 4 that the novel proposed undersampling achieves detailed image structures for undersampling factors up to 28. At least for seven-fold undersampling the image is visibly indifferent. The high acceleration factors were achieved by combining ideas of several acceleration techniques: a synthesis of RF-Encoding and Parallel Transmit can enable the non k-space acquisition and the sparsifying spaces, as they are used in CS, enable the measurement time reduction. Opposed to the post-processing that is performed by CS-schemes on randomly sampled k-space data, the herein presented scheme scans intentionally chosen excitations that are computed online depending on the pervious measurements. Since a single decision where to proceed is a simple threshold check, this can be implemented in real time using an efficient data-structure.

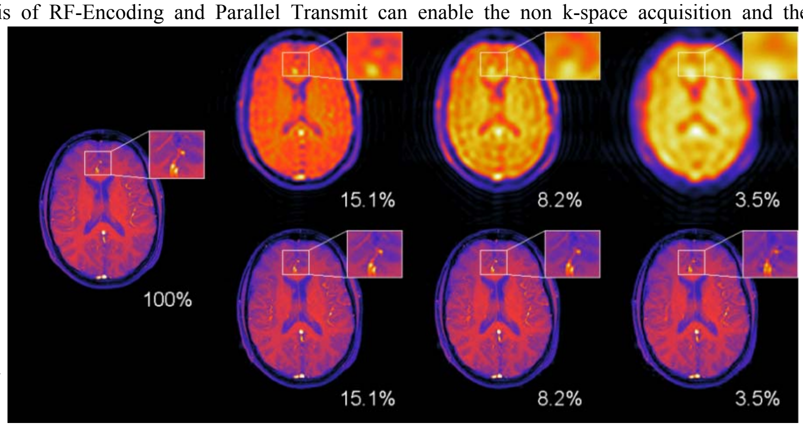


***Figure 4:*** *Images of a healthy volunteer's brain. The image on the left shows the reconstruction from full data. The first row shows images of various undersampling factors in k-space, while the images in the second row were reconstructed with the proposed scheme. The numbers in the lower right corner indicate the degree of undersampling.*

## Conclusion

A novel idea for acquisition time reduction beyond k-space sampling has been proposed. Theoretically evaluations show a high acceleration potential and should be encouraging for a technical implementation of image data acquisition directly in a sparse image domain using RF-encoding.

## Acknowledgement

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## References

1. Lustig, M., et al., MRM, 2007. **58**(6): p. 1182-95. 2. Akcakaya, M., et al., MRM, 2011. **66**(3): p. 756-67. 3. Katscher, U., et. al., MRM, 2010. **63**(6): p. 1463-70.
4.Schneider, J.T., et al., MRM, 2011. **65**(2): p. 409-21. 5. Buckheit, J., et al., *About WaveLab*. 2005.