

# A general analytical solution for optimized fat-suppression calculation

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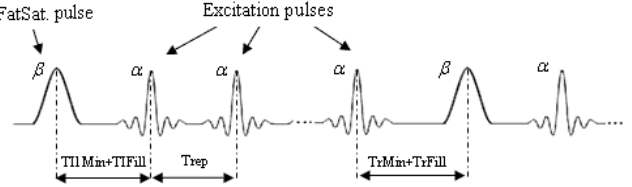
## Abstract

The effectiveness of spectral fat suppression depends on the optimal choice for the fat-suppression flip angle (conventional FatSat) or the inversion time (SPAIR and STIR) which vary with different acquisition sequences. One method [1] has been proposed to calculate the optimal FatSat flip angle in multi-slice experiments, but it does not cover the case of single-slab 3D acquisitions with multiple excitations between successive fat-suppression pulses, which is normally used in abdominal applications with short-TR spoiled GRE sequences. In this abstract, a general solution to calculate the optimized flip angle or the inversion time for spoiled GRE sequences is presented. The proposed solution can also be used in any other sequences (TSE etc.) where the transversal magnetization can be neglected before the next excitation.

## Background

In some applications, e.g. abdomen MRI, a short acquisition time is critical to avoid motion artifacts. For this purpose, a short TR and multiple excitations after each fat-suppression pulse are commonly used (see Fig.1). The optimum fat suppression is achieved if the longitudinal magnetization component of fat is null at the time point when the k-space center line is acquired.

**Fig.1.** Diagram of a spoiled GRE sequence with fat-suppression. The gradient pulses (slice selection, phase-encoding, spoiling gradients etc.) and the RF spoiling mechanism are not shown to simplify the figure.  $\beta$  -- Flip angle of the fat-suppression pulse.  $\alpha$  -- Flip angle of the excitation pulse. T11Min -- Minimum dwell time between  $\beta$  pulse and first following  $\alpha$  pulse. T11Min + TIFill = T11. Trep -- Time between successive  $\alpha$  pulses. TrMin -- Minimum dwell time between the last  $\alpha$  pulse after one  $\beta$  pulse, and the next  $\beta$  pulse. TrMin + TrFill = Tr.



## Method

We assume that the spoiling mechanism works perfectly, so that the magnetization in the x-y plane has no contribution to the following excitations. With this assumption, the magnetization expression of fat just before each RF pulse can be derived from the Bloch equations as following:

$$M_{\beta}(1) = M_0 \quad (1) \quad M_{\alpha}(1) = M_0 (1 - E_{T11}) + M_{\beta}(m) \cdot \cos(\beta) \cdot E_{T11} \quad (2)$$

$$M_{\alpha}(n) = M_0 (1 - E_{Trep}) + M_{\alpha}(n-1) \cdot \cos(\alpha) \cdot E_{Trep} \quad (3) \quad M_{\beta}(m+1) = M_0 (1 - E_{Tr}) + M_{\alpha}(N) \cdot \cos(\alpha) \cdot E_{Tr} \quad (4)$$

Where  $E_{T11} = \exp(-T11/T1)$ ,  $E_{Trep} = \exp(-Trep/T1)$ ,  $E_{Tr} = \exp(-Tr/T1)$ , and  $T1$  = longitudinal relaxation time of fat tissue,  $M_0$  = initial longitudinal magnetization of fat tissue,  $M_{\alpha}(n)$  = longitudinal magnetization of fat just before the  $n^{\text{th}}$   $\alpha$  pulse,  $M_{\beta}(m)$  = longitudinal magnetization of fat just before the  $m^{\text{th}}$   $\beta$  pulse,  $N$  = number of  $\alpha$  pulse between two  $\beta$  pulses.

In steady state, the longitudinal magnetization of fat before each  $\beta$  pulse should be the same, and this longitudinal magnetization component should be decreased to null when the k-space center line is excited, so that the optimized fat-suppression can be achieved. These two conditions can be described as following:

$$\text{Steady state condition: } M_{\beta}(m+1) = M_{\beta}(m) \quad (5) \quad \text{Optimized fat-suppression condition: } M_{\alpha}(\text{KSpaceCenterLine}) = 0 \quad (6)$$

To derive the formulas to calculate the optimum flip angle  $\beta$  for the FatSat pulse, and TIFill or TrFill for SPAIR (or STIR) pulse, the expression(1)-(4) is simplified as

$$M_{\beta}(1) = 1 \quad (7) \quad (\text{Since } M_0 \text{ will be balanced out during the calculation, it can be set to 1.})$$

$$M_{\alpha}(1) = A + B \cdot M_{\beta}(m) \quad (8) \quad M_{\alpha}(n) = C + D \cdot M_{\alpha}(n-1) \quad (9) \quad M_{\beta}(m+1) = E + F \cdot M_{\alpha}(N) \quad (10)$$

Where  $A = 1 - E_{T11}$ ,  $B = \cos(\beta) \cdot E_{T11}$ ,  $C = 1 - E_{Trep}$ ,  $D = \cos(\alpha) \cdot E_{Trep}$ ,  $E = 1 - E_{Tr}$ ,  $F = \cos(\alpha) \cdot E_{Tr}$

Combining expression(7)-(10) and the conditions (5)-(6), a closed analytical equation, which can be used to calculate  $\beta$  for the FatSat pulse, and TrFill or TIFill for SPAIR (  $Tr = TrMin + TrFill$ ,  $T11 = T11Min + TIFill$  ), is derived:

$$\{E + F \cdot C \cdot (D^{N-1}) / (D-1) + F \cdot A \cdot D^{N-1}\} / (1 - F \cdot B \cdot D^{N-1}) = \{C \cdot (D^{KSpaceCenterLine-1}) / (D-1) + A \cdot D^{KSpaceCenterLine-1}\} / (-B \cdot D^{KSpaceCenterLine-1}) \quad (11)$$

For a spoiled GRE sequence with FatSat pulse, Trep,  $\alpha$ , KSpaceCenterLine,  $N$ ,  $Tr = TrMin$ ,  $T11 = T11Min$  (  $TrMin$  and  $T11Min$  is determined by the RF pulse and gradient pulse duration ) are known. The only unknown variable is  $\beta$ . Based on equation (11),  $\beta$  can be solved as  $\beta = \arccos(B / E_{T11})$ , where

$$B = G1 / (G1 \cdot G4 - G2 \cdot G3), G1 = C \cdot (D^{KSpaceCenterLine-1}) / (D-1) + A \cdot D^{KSpaceCenterLine-1}, G2 = D^{KSpaceCenterLine-1}, G3 = E + F \cdot C \cdot (D^{N-1}) / (D-1) + F \cdot A \cdot D^{N-1}, G4 = F \cdot D^{N-1}.$$

For a spoiled GRE sequence with SPAIR pulse, Trep,  $\alpha$ ,  $\beta(180^\circ)$ , KSpaceCenterLine,  $N$ ,  $TrMin$ ,  $T11Min$  are known. The unknown variable is TIFill or TrFill. So the solution for SPAIR will include 2 steps as following:

Step A: Keep  $TrFill = 0$ , i.e.  $Tr = TrMin$ , calculate TIFill.  $TIFill \geq 0$  means the calculation is successful ( $TIFill \geq 0$ ,  $TrFill = 0$ ), while  $TIFill < 0$  or the calculation fails means we have to go to Step B.

Step B: Keep  $TIFill = 0$ , i.e.  $T11 = T11Min$ , calculate TrFill.  $TrFill \geq 0$  means the calculation is successful ( $TIFill = 0$ ,  $TrFill \geq 0$ ), while  $TrFill < 0$  or the calculation fails means there is no solution for this parameter set, i.e. the time to k-space center takes too long even when inverting a fully recovered fat signal.

The result formulas of the steps A and B are not shown in this abstract due to the limited space. But they can be easily solved from equation (11) with the above known information.

## Results and discussion

All the images below are acquired in breath-hold (15~19s) with a short-TR spoiled 3D GRE sequence on a Siemens MAGNETOM Verio 3.0T system (Erlangen, Germany). The acquired data matrix was  $320 \times 190 \times 64$  samples including partial Fourier in phase encoding and partition and parallel imaging (GRAPPA x2). 32 lines were acquired per FatSat or SPAIR pulse with  $\alpha=9^\circ$  and  $Trep=3.8$  ms. The calculated FatSat flip angle based on this particular sequence parameter set is  $153^\circ$  (Fig.2-A), and the calculated SPAIR TrFill and TIFill are 0 and 62.3ms respectively (Fig.2-B). Compared with Fig.2-C (reference image without fat suppression), the low intensity in the fat area (marked with red arrows) in Fig.2-A and Fig.2-B shows that the presented solution is working well for both FatSat and SPAIR.

The proposed solution is derived based on the Bloch equations (7)-(10) of the fat signal evolution, and two boundary conditions (5)-(6). It is analytical, accurate and can be commonly used, not only for the spoiled GRE sequence, but also for any other sequence type if the transversal magnetization can be neglected before the next excitation.

**Fig. 2.** In-vivo test for the presented solution.

- A. Fat suppression with FatSat pulse. Calculated FatSat flip angle =  $153^\circ$ .
- B. Fat suppression with SPAIR pulse. Calculated TrFill = 0, TIFill = 62.3ms.
- C. Reference image with fat signal. All acquisition parameters are the same with case A and B except that fat suppression is switched off to show the fat signal.



## References

[1] Method for optimizing fat suppression using the chemical shift selective MR imaging technique, US patent 6272369