SENSE imaging using the weak and strong voxel approach without the assumption of voxel functions being Dirac distributions

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Introduction: In the commonly applied Cartesian SENSE reconstruction [1], from an undersampled data set with reduced FOV the aliasing is eliminated by decomposing each set of aliased voxels separately. The advantage of this approach is the simple and fast calculation of the unaliased images. However, this method is only one possible reconstruction of the general theory of sensitivity encoding. Part of the reconstruction is modeling the applied voxel functions to ideal ones. Commonly applied SENSE imaging is based on two assumptions: 1) The applied and the desired, ideal voxel functions fulfil the orthonormality relation. This approach is called the weak voxel condition. 2) The desired voxel functions are Dirac distributions. The first assumption is only valid if the product of the reconstruction matrix F and the encoding matrix E is identity. Otherwise, the voxel quality is reduced. The second assumption leads to residual aliasing especially when the coil sensitivities rise very sharply [2, 3]. This artifact scales with the voxel size and therefore becomes more attenuated if k-space is partially zerofilled. Because of the numerically demanding reconstruction, in previous work SENSE reconstruction was either applied with the weak or the strong voxel approach assuming Dirac distributions as voxel functions [1-4]. In this work, the theoretical framework for the application of sensitivity encoding without using the assumption of voxel functions being Dirac distributions is introduced and first results of this new method in in-vivo imaging are presented.

Theory: In the general theory of sensitivity encoding [1] the reconstructed image $\mathbf{v} = F\mathbf{m}$ is calculated from the undersampled k-space data $\mathbf{m}$ using the reconstruction matrix F. In the strong voxel approach the reconstruction matrix $F = E^H C^{-1}$ is determined by the encoding matrix E and by the correlation matrix of the encoding functions C. In the generalized weak voxel approach only the encoding matrix is required $F = (E^H \Psi^{-1} E)^{-1} E^H \Psi^{-1}$ with Ψ being the sample noise matrix. The elements of the matrices E and C require the calculation of integrals which may be discretized when assuming the voxel functions being Dirac distributions. In contrast, approximating the coil sensitivities $s_\gamma(x) = \sum_{l=0}^{M} c_{\gamma,l}(x - x_0)^l$, which are chosen one-dimensional for simplicity, using polynoms of degree M, the encoding matrix and the correlation matrix of the encoding function may be calculated analytically:

$$E_{(\gamma,\kappa),\rho} = \sum_{l=0}^{M} c_{\gamma,l}(-i)^l \frac{d^l}{dk^l}(\mathrm{sinc}(0.5k)e^{ikx})\Big|_{k=k_\kappa} \quad \text{and} \quad C_{(\gamma,\kappa)(\gamma',\kappa')} = \sum_{l=0}^{M} c_{\gamma,l}(-i)^l N \frac{d^l}{dk^l}(\mathrm{sinc}(0.5Nk)e^{-0.5ik})\Big|_{k=k_\kappa - k_{\kappa'}}.$$

Using these formulas for the calculations of the elements of E and C allows an numerically efficient way of applying the weak and strong voxel approach of SENSE imaging without assuming Dirac distributions as voxel functions.

Methods: In-vivo data were acquired on a 3T scanner (Siemens Magnetom Verio) using a TSE-sequence (T_R=3000 ms, T_E=17 ms, flip angle 120°, turbo factor 5, slice thickness 3 mm, matrix 320x320, FOV 220 x220 mm²) and a 4 channel receiver coil. Reconstruction was performed with the SENSE method using the strong voxel approach (sva), the weak voxel approach (wva) and the weak voxel approach assuming Dirac distributions as voxel functions (conv), which is the conventional SENSE technique. Coil sensitivity maps were determined by polynomial fitting (M=5) from 32 central k-space lines. These sensitivity maps were used for all reconstruction methods. Both the full data set and a twofold accelerated data set generated by discarding every second line from the full data set were reconstructed. As the ill-conditioned matrices in sva and wva had a well-determined numerical rank an adapted TSVD was applied finding the gap in the singular value spectra [5].

Results: In Figure 1**a-f** the results of the different SENSE methods applied to the full data set are presented. The images of the new SENSE techniques show a different texture than conventional SENSE imaging. This can be also seen from the difference images in Figure 2 (first row). Additionally, the aliasing behavior of the different methods can be seen in Fig. 1**g-l**. The residual aliasing is calculated in Figure 2 (second row).

Discussion: Residual aliasing can appear in the conventional SENSE method because the voxel functions are assumed to be Dirac distributions [2, 3]. It was shown that by approximating the coil sensitivity maps with polynomials the required reconstruction matrices of generalized SENSE imaging can be calculated numerically efficient. First results of this new reconstruction method were demonstrated. The generalized weak voxel approach can increase the reconstruction accuracy. The strong voxel approach may provide the highest reconstruction accuracy but is less efficient in reconstruction time.

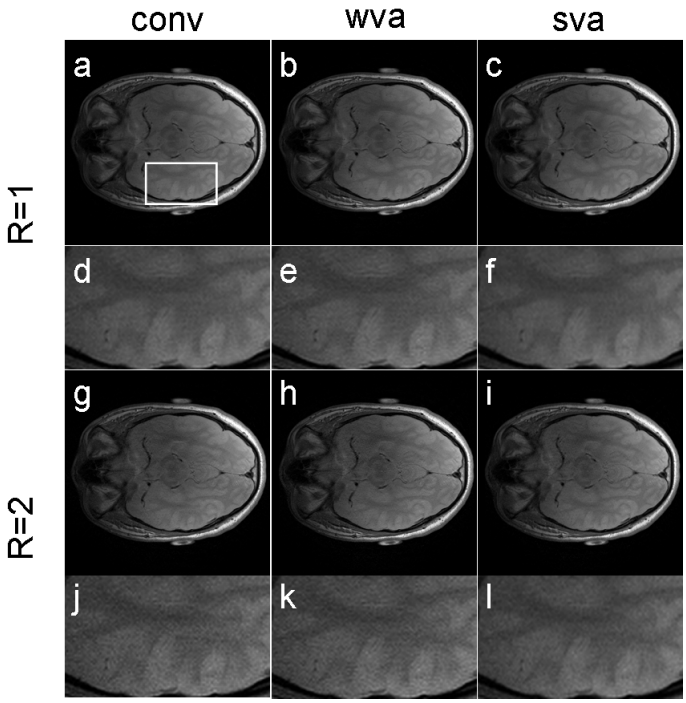

Figure 1: Images from SENSE using the different voxel functions conv, wva and sva for acceleration factor R=1 (1st and 2nd row) and R=2 (3rd and 4th row). The white box in **a** demonstrates the magnified regions shown below the images with full FOV.

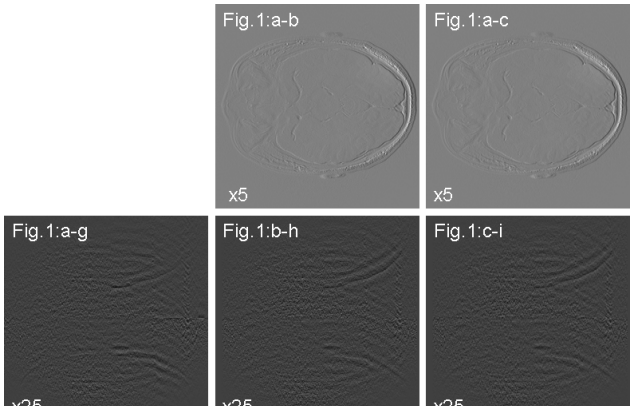

Figure 2: In the first row difference images between conventional and the proposed SENSE methods are shown. For better presentation the images were scaled by a factor of 5 in comparison to the images in Figure 1. The differences between the unaccelerated and accelerated images for the different methods are scaled by a factor of 25.

References: [1] Pruessmann KP, et al. Magn Reson Med 1999, 42: 952-962.
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