Nonlinear Formulation of the Magnetic Field to Source Relationship for Robust Quantitative Susceptibility Mapping

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Introduction: In quantitative susceptibility mapping (QSM), the susceptibility distribution is inverted from the magnetic field estimated from MR phase images [1,2,3]. However, the phase values are not always reliable due to noise and phase unwrapping errors. A proper treatment of the noise in the data is an unsolved technical problem in QSM. We propose to solve this problem by using a noise model attuned to the complex MR signal. QSM is obtained by solving an inverse problem where the data term is nonlinear. As part of the proposed nonlinear approach, we also develop a nonlinear field map estimation method. Compared to the previously developed phase-based linear methods, the nonlinear method demonstrated a marked improvement in reconstructing QSM for intracranial hemorrhages.

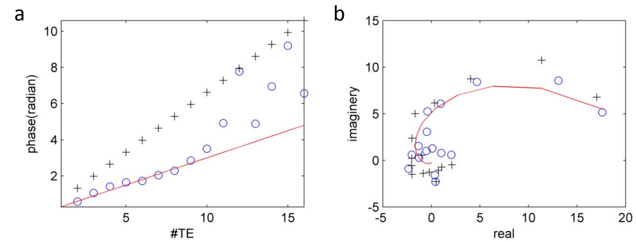

Fig. 1. a) The fitted phase (black cross) failed to catch the true phase (red line) because the measured phase (blue circles) are corrupted by noise. b) The noise corruption problem was reduced in the complex plane used in the nonlinear fitting method.

Theory: The field map estimation problem is formulated as

$$f^*, \phi_0^* = \text{argmin}_{f,\phi} \sum_j \| s(TE_j) - A(TE_j)\exp[i(f\times TE_j + \phi_0)]\|_2^2.$$

Here, f is the frequency shift in a voxel, s the complex MR signal measured at multiple TEs, ϕ_0 is an initial phase, j denotes the jth echo, and A is the signal magnitude in the absence of noise. The susceptibility χ is calculated by minimizing the structural difference between the susceptibility and prior anatomical information combined with a nonlinear data fidelity constraint:

$$\chi^* = \text{argmin}_\chi \|MG\chi\|_1, \text{ subject to } \|W(\exp(iD\chi) - \exp(if))\|_2^2 = \mu,$$

where M is the anatomical prior, G is the gradient operator, W is a diagonal weighting matrix reflecting the voxel-by-voxel reliability of the estimated field map f, D is a matrix performing the dipole convolution and μ is the expected noise level. .

Materials and Methods: *Algorithm implementation*. The nonlinear least square fitting problems in the field map estimation and in the dipole inversion were solved in an iterative manner. In each iteration, the nonlinear exponential functions were linearized by using Taylor expansion and keeping only the zeroth and first order terms. Then, the linear problems were solved using QR decomposition for field map estimation and conjugate gradient for the dipole inversion. The weighting W was adjusted to account for model error in each iteration of the nonlinear solver based on the voxel-by-voxel residual of the previous iteration, and we termed this method model error reduction through iterative tuning (MERIT). Other detailed implementation methods were published in [3,4]. *Numerical simulation for field map estimation*. Complex MR signals were simulated at different TEs assuming a known T2* decay and frequency shift. The simulation was repeated 1000 times with freshly generated Gaussian noise added to real and imaginary components. Differences between the true and estimated frequencies were recorded. *Patient scans*. Two QSMs were obtained for each of 13 de-identified patient cases with intracranial hemorrhages using the linear and nonlinear inversion methods. Imaging parameters were: TR=45ms, 8 TEs evenly spaced between 5ms to 40ms, bandwidth = 244Hz/voxel, field of view = 24cm, slice thickness=2mm, acquisition matrix=240×240×28~54 reconstructed to 256×256×28~54, scan time~5min. QSM images were scored by a neuroradiologist based on the degree of artifacts with 0=none and 4=severe. Image score differences were assessed with a Wilcoxon signed-rank test.

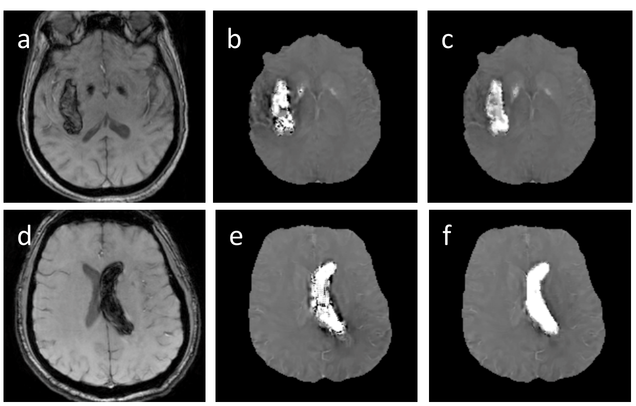

Fig. 2. QSM in two patients a&d) magnitude images of the hemorrhage. b&e) linear inversion. c&f) nonlinear inversion.

Results: Improvement of the field map estimation is shown in Fig. 1. In the 1000 repetitions, neither method showed significant bias in the field map estimation (mean=-.39Hz and -0.12Hz, P=0.44 and 0.42 for linear and nonlinear fittings, respectively). However, the standard deviation of the difference was markedly reduced in the nonlinear fitting (std = 16.1Hz and 4.8Hz for linear and nonlinear fittings, respectively). Two exemplary hemorrhage cases are shown in Fig. 2. The nonlinear inversion method showed decreased noise inside the hemorrhage and removal of the shadowing artifact. The average artifact score of linear QSM was 2.6±1.1 (mean±standard deviation) while that of the nonlinear method was 0.5±0.7. The artifacts scores in nonlinear QSM were lower than linear QSM in all cases and the difference was statistically significant (P<0.001).

Conclusion: In this study, we proposed a nonlinear QSM reconstruction method in which phase noise is properly accounted by considering the complex signal instead of the phase, and model error is reduced by iterative tuning. Numerical simulation and patient data showed significant improvement in regions with low signal to noise ratio. The enhanced image quality is especially helpful for imaging intracranial hemorrhages.

References: [1] Wharton et al. MRM:63(5):1292-1304; [2] Liu et al. MRM:61(1):196-204; [3] Liu et al. Neuroimage; *in press*; [4] Liu et al. Proc Int Soc Magn Reson Med 2011.p2737.