Analytical Description of Magnetization Transfer Effects on the Transient Phase of Balanced SSFP

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Introduction. Inversion recovery (IR) balanced steady-state free precession (bSSFP) [1] has been proposed for fast T_1, T_2, and spin density quantification [2,3]. Recently, it has been realized that magnetization transfer (MT) effects can lead to considerable deviations in the calculated T_1 and T_2 values from IR-bSSFP experiments with short RF pulses [4,5]. In this work, an analytical expression for the IR bSSFP signal recovery is derived that takes into account MT effects. Simulations of the Bloch equations are used to confirm the derived expression. This work forms a basis for a possible fast quantitative analysis of MT effects with 2D IR bSSFP.

Theory. After inversion and initial $\alpha/2$ – TR/2 magnetization preparation, the transient magnetization $\mathbf{F}_n = \mathbf{M}_n - \mathbf{M}_{SS}$ is observed during its approach to the steady state [6,7]. Here, $\mathbf{M}_n$ represents the magnetization at TE = TR/2 and after n excitation pulses and $\mathbf{M}_{SS}$ denotes the steady state magnetization. The transition matrix **T** between successive magnetization vectors is described by a relaxation matrix **E,** two rotation matrices $\mathbf{R}_y$ and $\mathbf{R}_z$ and an exchange matrix **A** to take into account MT effects that depend on the restricted pool fraction F and the forward exchange rate k_f [8], according to

$$\mathbf{T} = \mathbf{A}(TR)\cdot\mathbf{E}(TR/2)\cdot\mathbf{R}_z(\pi)\cdot\mathbf{R}_y(\alpha)\cdot\mathbf{E}(TR/2) \qquad (1)$$

The transient response of MT-IR bSSFP can now be found by calculating the eigenvectors of the transition matrix **T**: $\mathbf{TF}_n = \lambda\mathbf{F}_n$ [6]. The real eigenvalue λ corresponding to the decay rate of the transient signal parallel to the steady state magnetization of the two-pool bSSFP model is given by:

$$E_1^* = \frac{1}{3}\left[K + A^{-1} + A\right], \text{ where } A := \frac{1-i\sqrt{3}}{\sqrt[3]{16}}\sqrt[3]{M + i\sqrt{4L^3 - M^2}} \qquad (2)$$

$$\text{and } K := ab + (ag - E_2)\cos\alpha,\ L := K^2 + 3\left(agE_2 + (abE_2 - h)\cos\alpha\right),$$

$$M := -2a^3b^3 - 9a^2bE_2g + 27E_2h + 3(E_2 - ag)(abE_2 + 2a^2bg - 3h)\cos^2\alpha$$
$$+ 2(E_2 - ag)^3\cos^3\alpha - 3a\left(2a^2b^2g + aE_2(b^2 + 3g^2) - 3(E_2{}^2g + bh)\right)\cos\alpha,$$

with definitions: $a := E_1/(1+F),\ b := (F + f_k)f_w,\ g := 1 + Ff_k,\ h := E_1{}^2 f_k f_w,$

$$f_k := \exp(-(k_f + k_r)TR),\ f_w = \exp(-\langle W(\Delta \to 0)\rangle T_{RF}),\ M_0 = M_{0,f}$$
$$E_1 \equiv E_{1,f} \equiv E_{1,r} := \exp(-TR/T_{1,f}),\ E_2 \equiv E_{2,f} := \exp(-TR/T_{2,f})$$

In good approximation, the IR bSSFP signal at t = 0 can be described by $S_0 = -M_0\sin(\alpha/2)$, while the two-pool bSSFP steady-state signal is given by [8]. From this $INV = 1 - S_0/S_{SS}$ can be calculated and the exponential time course of the two-pool IR bSSFP signal can be described by

$$S_n(TR)/S_{SS} = 1 - INV\cdot e^{-nTR/T_1^*} \qquad (3)$$

Methods. *Simulations:* For verification of Eq. (2), simulations of the IR bSSFP two-pool Bloch equations [9] were performed. To this end, sinc-shaped RF pulses with a fixed time-bandwidth product of 5.4 but variable duration were used. Deviations from the ideal slice profile were accounted for by calculating an effective flip angle α_{eff}. The mean saturation rate <W> of restricted pool protons were calculated as a function of α_{eff} and T_{RF} according to [8]. Simulations were performed with either long (T_{RF} = 3.61ms, corresponding to “noMT”) and short pulse duration (T_{RF} = 570ms, corresponding to “MT”). Quantitative MT parameters F and k_f were taken from [8]. *Measurements:* Experiments were performed on a 1.5T clinical system. In vivo axial IR 2D bSSFP human brain scans (1mm in-plane resolution, 8mm slice thickness) were performed using a segmented imaging sequence with a flip angle of α = 40°, a TR of 6.64ms and a receiver bandwidth of 1150Hz/Pixel. After adiabatic inversion, 31 images were acquired with equally distributed inversion times between 210ms and 3210ms. One data set with long (noMT) and one with short RF pulse (MT) were acquired. The first data set with T_{RF} = 3.61ms was used to fit T_1 and T_2 in two regions of interest (Fig. 2) using the standard IR bSSFP equation [3].

Results & Discussion. *Simulations:* Fig. 1 shows that the derived two-pool IR bSSFP equation corresponds to numerical simulations of the two-pool Bloch equations. Thus, the matrix approximation in the derivation of the decay rate seems appropriate. *Measurements:* The relaxation times obtained from a fit to the MT-free data points were T_1 = 812ms and T_2 = 51ms for white matter (WM) and T_1=1183ms and T_2 = 57ms for gray matter (GM). A comparison of both measured time series with the two-pool IR bSSFP signal equation is given in Fig. 3. Correspondence for the short pulse (red curves) is not exact and has to be further investigated.

Conclusion. An analytical description of MT effects in the transient phase of bSSFP has been introduced and compared to numerical simulations and measurements in brain white and gray matter. The new equation has a high potential for derivation of quantitative MT parameters using fast 2D IR bSSFP scans [5], such as F and k_f, from only two measurements, one without MT and one with MT effects.

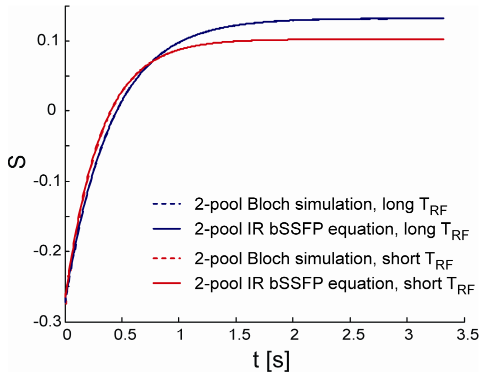

Fig. 1: IR bSSFP two-pool Bloch simulation vs. analytical solution (Eqs. 2,3) for brain white matter(T_1=812ms and T_2=51ms from fit with standard IR bSSFP equation [3],and F and k_f from [8]).

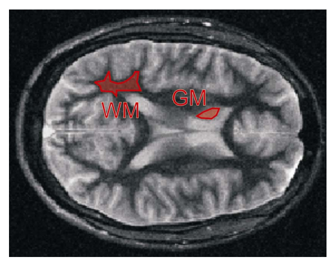

Fig. 2: Exemplary IR bSSFP image displaying white matter (WM) and gray matter (GM) regions.

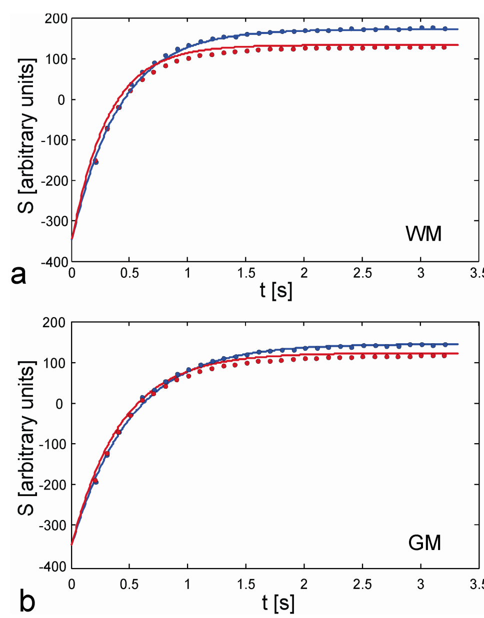

Fig. 3: Measured IR bSSFP signal (dots) and two-pool IR bSSFP signal (solid lines) using Eqs. 2,3. Blue curves: T_{RF} = 3.61ms, no MT. Red curves: T_{RF} = 570ms, MT.

References. [1] Deimling et al., Proc. ISMRM 2 (1994); [2] Scheffler et al., MRM 45(4):720-723 (2001); [3] Schmitt et al., MRM 51(4):661-667 (2004); [4] Blumhagen et al., Proc. ISMRM 17 (2009); [5] Ehses et al., Proc. ISMRM 18 (2010); [6] Scheffler, MRM 49(4):781-783 (2003); [7] Hargreaves et al., MRM 46(1):149-158 (2001); [8] Gloor et al., MRM 60(3):691–700 (2008); [9] Sled et al., MRM 46(5):923–31 (2001).