A Novel Compressed Sensing Approach to Accelerated Quantitative MRI Using Model-Driven Adaptive Sparsifying Transforms

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Introduction: Quantitative MRI (qMRI) provides numerical maps of MR parameters, such as T1 and T2 relaxation times,, that help to glean additional information about tissue micro structure compared to the traditional visualization of human anatomy by MRI. The estimation of qMRI parameters requires acquisition of multiple datasets at different values of pulse sequence (control) parameters, e.g. flip angles or inversion times, followed by reconstruction of the corresponding series of images and fitting these images to an analytical signal model equation to yield parametric maps of interest. As a result, qMRI typically incurs a several-fold increase in scan time, making it prone to patient motion and limiting its utility in clinical settings. Recently, a number of advanced image reconstruction techniques have been proposed [1-3] to obtain parameter maps from accelerated undersampled data that rely heavily on theoretical knowledge about signal evolution in the parametric dimension and its dependence on control parameters. In [2], the signal equation is used to design a representation system that should allow for a sparse representation of an image series, while [3] uses the fact that signal curves can be approximated well by piecewise constant or linear functions to design a sparsifying transform acting in the parametric dimension. Although both approaches have demonstrated promising results, the generality of the sparsifying transforms they use limits the achievable acceleration factors In this work, we propose a model-driven compressed sensing (CS) approach that interleaves signal estimation with adaptive update of sparsifying transforms based both on the analytical model equation and signal estimates.

Theory and Methods: Let f be a parametric image series dependent on user-prescribed control parameters p_{ctrl} and unknown free parameters p, which are governed by the analytical model $F(p_{ctrl};p)$. In accelerated imaging, f has to be obtained from an underdetermined problem $Ef=b$, where E is the encoding matrix and b is the measured *k*-space data for all values of control parameters. Suppose that we have a good estimate $\tilde{f}$ of signal behavior in parametric dimension and can design a transform $\Phi_{\tilde{f}}$ so that $\Phi_{\tilde{f}}f$ is piecewise constant. Then f can be reconstructed accurately by solving the regularized problem $f = \arg\min_f \left(\|Ef-b\|_2^2 + \lambda\|\Delta_p^1\Phi_{\tilde{f}}f\|_{l_2}\right)$, where Δ_p^1 is the 1st difference operator in the parametric dimension and λ is a regularization parameter. For example, if $\tilde{f}$ is a good approximation to the solution f, then the choice of $\Phi_{\tilde{f}} = diag(1/\tilde{f})$ will result in a nearly constant function and, combined with the 1st difference operator, will provide a good sparsification. Intuitively, the goal of $\Phi_{\tilde{f}}$ is to "straighten" the signal curve prior to an application of Δ_p^1. Since no good estimate of f is available initially, we propose to utilize analytical model to adjust sparsifying transform adaptively in a series of iterations as described in the Algorithm Outline. The sparsifying transforms are updates until $\|\tilde{f}^{(n)} - \tilde{f}^{(n-1)}\|_2 < \varepsilon$, and f is obtained using conjugate gradient (CG). In practice, 10 transform updates and 100 CG iterations were needed keeping total reconstruction time on the order of several minutes.

Algorithm Outline
Initialize: $n=1, \tilde{f}^{(0)} = Id$
Step1: Update solution vector: $f^{(n)} = \arg\min_f \left(\|Ef-b\|_2^2 + \lambda\|\Delta_p^1\Phi_{\tilde{f}^{(n-1)}}f\|_{l_2}\right)$
Step 2: Estimate parameters by fitting to the analytical model: $p^{(n)} = \arg\min_p \|F(p_{ctrl};p) - f^{(n)}\|_2^2$
Step 3: Reproject from parametric space to image space: $\tilde{f}^{(n)} = F(p_{ctrl};p^{(n)})$
Go to Step 1 with updated sparsifying transform $\Delta_p^1\Phi_{\tilde{f}^{(n)}}$

We apply the proposed algorithm to T1 mapping using variable flip angle (VFA) [4] and Look-Locker inversion recovery [5] methods. A realistic brain phantom [6] was used to simulate a VFA acquisition with TR=8 ms and 10 flip angles uniformly distributed from 1° to 19°. Eight-fold accelerated *k*-space data were generated for random sampling with 8 coil receivers both with and without addition of white Gaussian noise. Three 128x128x10 image series were reconstructed using iterative SENSE [7], CS with 1st difference in parametric dimension regularization [3], and the proposed algorithm, and T1 maps were obtained from each series.

Fully sampled inversion recovery data from a healthy volunteer were collected for 40 inversion time increments of 72 ms with TR=3 s on a 1.5 T clinical scanner (Philips Healthcare) . The single channel data were retrospectively undersampled by taking random phase encodes with acceleration factor 6. Reconstruction results for 224x224x40 image series and T1 maps were compared for the proposed method, CS with 1st difference and zero-filling.

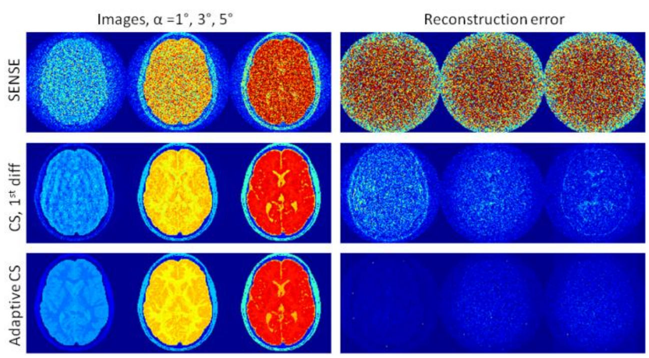

Figure 1. Representative images from parametric VFA image series and their reconstruction error for different algorithms.

Results: In the absence of noise the proposed algorithm provides near perfect reconstruction of T1 maps (not shown) with normalized root mean square error (nRMSE) of 0.4%, while SENSE and CS suffer from some resolution loss with nRMSE = 1.25% and 5.54%, respectively. Figure 1 compares results of image reconstruction of simulated noisy VFA data. Note the superior noise properties of the proposed adaptive algorithm. Errors from individual images propagate into parametric T1 maps resulting in nRMSE of 32.92% (SENSE), 5.82% (CS) and 4.32% (adaptive CS).

Figure 2 compares reconstruction results for R1=1/T1 maps. The use of the adaptive sparsifying transform helps minimize residual aliasing artifacts still present in CS results, which is reflected in nRMSE of 15% for zero-filling, 9.4% for CS and 8.0% for adaptive CS.

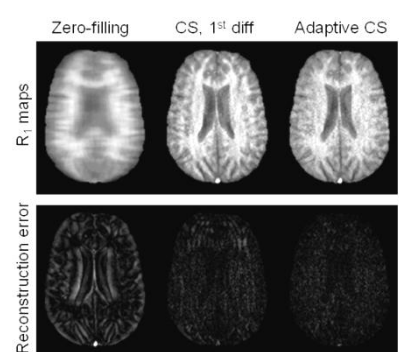

Figure 2. R1 maps for acceleration factor of 6.

Conclusions: Inter-image dependencies in parametric image series can be exploited in CS-like reconstruction to accelerate qMRI. However, when the actual data do not conform to the chosen sparsifying strategy, reconstruction accuracy is compromised or acceleration is limited. The proposed adaptive model-driven design of sparsifying transform based on available signal estimates further enhances sparsity and improves accuracy of parametric map reconstruction as demonstrated in T1 relaxometry. The proposed algorithm can also be used in other qMRI applications, especially when the analytical model allows an efficient fit to facilitate computations for sparsifying transform updates such as in case of VFA T1 mapping and spin echo T2 mapping. The proposed algorithm presents a computationally inexpensive alternative to more complicated and time consuming methods that utilize analytical models in the parametric reconstruction, such as [1].

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