Comparison of Wavelet Subband Decomposition Methods for High-Frequency Subband CS

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Introduction: Compressed Sensing (CS) is an acquisition and reconstruction technique that can reduce the measurement size without causing aliasing artifacts [1,2]. High-frequency Subband (HiSub) CS has been developed to efficiently combine both parallel imaging and CS by exploiting wavelet-domain sparsity and the wavelet tree structure [3]. One key operation for HiSub CS is the wavelet subband decomposition, which allows selective application of different undersampling patterns and reconstructions in different k-space regions. In this study, we describe two wavelet decomposition methods and apply them to high-resolution T1- and T2-weighted 3D breast imaging.

Theory: Wavelets are localized in both time (image domain) and frequency (k-space), and the wavelet filters in k-space can be demodulated by the wavelet decomposition (Fig 1). The wavelet decomposition transforms full k-space data into the local k-space regions (u_{LH}, u_{HL} and u_{HH}) that directly correspond to the wavelet subbands (w_{LH}, w_{HL} and w_{HH}) and allows independent estimation of each localized k-space region. To better exploit wavelet-domain sparsity, HiSub CS employed CS for only high spatial frequencies (or high-frequency subbands) while parallel imaging is used for low-spatial-frequencies.

Wavelet aliasing should be removed to separate full k-space data into the local k-space regions, and we describe two ways to eliminate the wavelet aliasing. **Type 1 (approximate method):** a simple approach is to divide the full k-space data by each wavelet spectral weighting and select each local region. This does not require any restrictions on undersampling patterns, but residual wavelet aliasing may still exist. **Type 2 (exact method):** a more complicated approach is to use the fact that discrete wavelet transforms include down sampling (replications in k-space). The wavelet aliasing can be completely removed by solving an equation constructed by quadruplets of k-space locations. This method, however, requires quadruplet sampling, whereby each sampling location is 'replicated' in 4 quadrants of k-space [3].

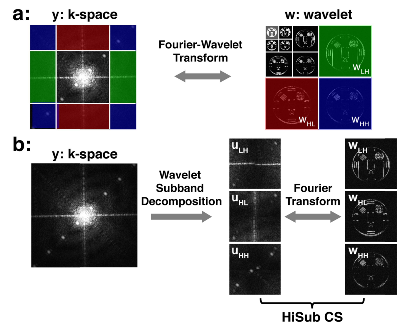

Fig 1: Wavelet decomposition for HiSub CS.

Methods: Imaging experiments were performed on a 3.0T GE MR750 system in a total of 6 subjects (3 T1-weighted and 3 T2-weighted scans). A custom-fitted 18-channel breast coil array was used, enabling parallel imaging in two dimensions (L/R and S/I) and providing high SNR [4]. We used the normalized root mean squared error (nRMSE) by the maximum signal to evaluate the differences between fully sampled and reconstructed images. HiSub CS was performed separately on each coil, and the root sum-of-squares was used to combine all the individual coil images. We used the Cohen-Daubechies-Feauveau (CDF) 9/7-wavelet transform.

For T1-weighted imaging, a 3D SPGR sequence was used. Elliptical full k-space data sets (matrix size=252×488 ×152) were acquired, and 4 different reconstructions (ARC, HiSub CS-Type1, HiSub CS-Type2 and L1-SPIRiT [5]) were retrospectively applied. The variable density Poisson-disk sampling is used for L1-SPIRiT as an example of a "standard" CS sampling/reconstruction approach.

For T2-weighted imaging, a 3D FSE sequence [6] was used in combination with parallel imaging (3×2; k_y×k_z). Only the HiSub CS-Type1 method was applied since the regularly undersampled parallel imaging data sets (matrix size=512×320×152) were not compatible with the quadruplet and Poisson-disk sampling patterns.

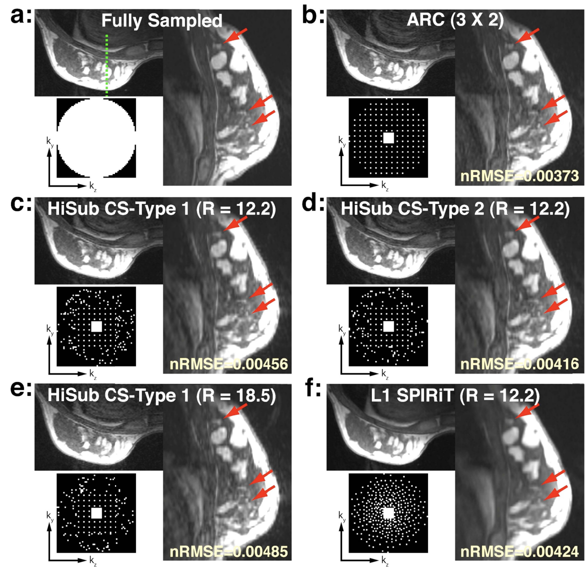

Fig 2: T1-weighted images. (a) Fully sampled, (b) ARC (R=3×2), (c) HiSub CS-Type1 (R=12.2), (d) HiSub CS-Type2 (R=12.2), (e) HiSub CS-Type1 (R=18.5), and (f) L1-SPIRiT (R=12.2). Note that the arrows indicate fine breast structures.

Results and Discussion: Fig 2 shows an example of T1-weighted images. For outer k-space (R_{CS} = 16), HiSub CS-Type1 used random undersampling, and HiSub CS-Type2 used quadruplet random undersampling. Both Type1 and 2 used the same regular undersampling for inner k-space (R_{PI} = 6). HiSub CS-Type2 has the lowest nRMSE of 0.00416, followed by L1-SPIRiT (0.00424) and Type1 (0.00456). However, depiction of fine structures was limited in L1 SPIRiT (see the arrows in Fig 2f) while morphology on both HiSub CS-Type1 (Fig 2c) and Type2 (Fig 2d) images closely resembled the fully sampled and ARC acquisitions (Fig 2a and b). To assess the limits for acceleration, we used a net acceleration factor of 18.5, and HiSub CS-Type1 still maintained good imaging quality (Fig 2e). The results were consistent for all 3 cases.

Fig 3 shows an example of T2-weighted images using HiSub CS (R = 10.7). Random k-space samples were selected from parallel imaging k-space data, and HiSub CS-Type1 maintained excellent image quality in all three planes (nRMSE=0.00643).

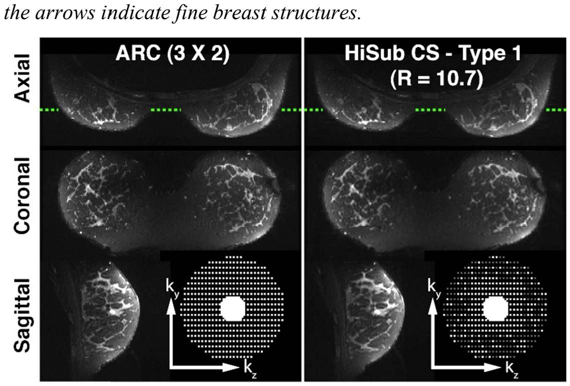

Fig 3: 3D T2-weighted imaging using ARC (R=3×2) and HiSub CS-Type1 (R=10.7).

Conclusion: HiSub CS formalizes a direct link between k-space and wavelet domains to apply separate undersampling and reconstruction for high- and low-frequency k-space data. Both wavelet decomposition methods (Type1 and 2) showed the improved reconstruction performance from using HiSub decomposition with high acceleration factors (R = 10.7 – 18.5). The exact decomposition had slightly better image quality and lower nRMSE while the approximate decomposition can be applicable for any undersampling pattern.

Reference: [1] Donoho, IEEE TIT, 2006;52:1289, [2] Candes et al., Inverse Problems, 2007;23:969, [3] Sung et al. ISMRM p70 (2011), [4] Nnewihe et al., MRM, 2011; 66:281, [5] Lustig et al. MRM 2010; 64:457, [6] Busse et al, ISMRM p1702 (2007)