Limits of Acceleration for Combinations of Compressed Sensing and Parallel Imaging

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INTRODUCTION: The combination of compressed sensing and parallel imaging (CS & PI) enables accelerated acquisition of MRI data by jointly exploiting image sparsity and coil sensitivity encoding using the idea of joint multicoil sparsity [1-3]. The limits of acceleration for each technique are known. Acceleration in compressed sensing is limited by image sparsity, with the number of required k-space samples being about 3-5 times the number of sparse coefficients [4-5]. Although in theory the maximum acceleration for parallel imaging depends on the number of coils, in practice, it is intrinsically limited by electrodynamic constraints [6-7]. However, the theoretical and practical limits of acceleration for the combined approach remain uncertain. In this work, we investigate the limits of acceleration for CS & PI using Monte Carlo simulations to characterize the performance of the combined reconstruction with respect to the number of coils for truly-sparse and compressible MR images. A complete set of electromagnetic (EM) fields is employed as a hypothetical optimal coil array, capable of achieving ultimate intrinsic SNR (UISNR) [6-7] for parallel imaging reconstructions.

THEORY: Optimal performance for CS & PI reconstruction can be obtained by using a different random undersampling pattern per coil, which will maximize incoherence along the coil dimension. However, it is inefficient in practice, since all the coils share the same gradients and therefore multiple sequential acquisitions would be required, loosing the advantages of parallel data acquisition and resulting in increased total acquisition time. In this work, we focus our analysis to the practical case of parallel data acquisition with the same random undersampling pattern for each coil. Even though the same random k-space undersampling pattern is shared for all coils, the convolution in k-space with the distinct coil sensitivities will generate intercoil incoherence, which approximates the behavior of using different random undersampling patterns per coil. To assess the limits of acceleration for the practical case, we modeled an array with an effectively infinite number of coils, using the same EM modes that allow calculating the best possible SNR for parallel imaging [8].

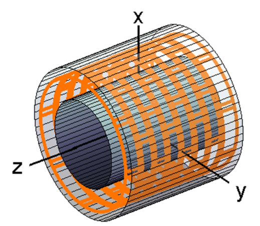

Fig. 1: Geometry set-up used for the simulation of coil sensitivities. Coils are defined on a cylindrical surface, surrounded by a conductive shield. This example shows the 128-element array.

METHODS: Full-wave coil sensitivities on a transversal slice (x-y plane) through the center of a 20 cm radius dielectric cylinder were simulated at 3 T for a number of arrays (N_c=8,16,32,64,128), using appropriated combinations of current basis functions [8], defined on a cylindrical surface 5 mm away from the sample and surrounded by a conductive shield (Fig. 1). The complete basis of current modes (15,714 modes) was employed to calculate as many coil sensitivities associated with the hypothetical optimal array that results in UISNR [8]. Planar coil arrays with different numbers of elements (N_c = 2, 4, 8, 16, 32) were simulated at 1.5 T, using the Biot-Savart law to compute coil sensitivities at voxel positions (512-point grid) along the axis of the cylinder. Coil sensitivities for the optimal array were calculated also in this case using the full-wave mode expansion. 1D simulations were performed assuming a truly sparse signal in the image domain with random positioning of the non-zero coefficients (number of points N = 512, number of nonzero points K = 32). 2D simulations were performed using a 64x64 brain image, which can be recovered with less than 5% error using its largest 290 Haar wavelet coefficients (sparsity K = 290). Two reconstruction cases were considered: (a) noise-free signal, to evaluate the required number of samples to achieve error-free reconstruction, and (b) signal with noise (SNR = 100), to evaluate the effect of the number of coils on the SNR. Acceleration was simulated by decimating the k-space representation of the multicoil data according to a random undersampling pattern. Reconstruction was performed by enforcing joint multicoil sparsity [3] with an orthogonal matching pursuit (OMP) algorithm for 1D simulations and iterative soft-thresholding for 2D simulations. The root mean square error (RMSE) over 100 different random k-space undersampling realizations was computed for different numbers of samples and coils.

RESULTS: 1D reconstructions of the truly sparse signal achieved error-free results for the noise-free case. The number of k-space samples to achieve error-free reconstruction decreased with the number of coils, with a lower bound given by the sparsity of the signal for the ultimate-SNR coil case (Fig. 2). 1D reconstructions with noise presented similar trends, but with a noise floor that decreased with the number of coils (not shown). 2D reconstructions of the compressible brain image presented a pseudo-noise floor given by the compression error. The number of required k-space sampled to reach the pseudo-noise floor also decreased with the number of coils and were ultimately bounded by the signal sparsity (Fig. 3). About 320 k-space samples were required for the optimal coil to achieve a reconstruction error within 10% of the pseudo noise floor, which are very close to the sparsity level K=290.

DISCUSSION: The minimum number of required k-space samples for joint CS & PI reconstructions is bounded by the number of sparse coefficients, which removes the oversampling factor of 3-5 for compressed sensing alone and approximates the theoretical bounds for ideal but impractical L_0-norm minimization described in the original compressed sensing publications [9]. This lower bound was found even for noise-free simulations, where standard parallel imaging reconstructions using the optimal array would have require only one k-space sample. The fact that even noise-free acquisitions require K samples per coil for joint CS & PI reconstruction may be due to amplification of noise-like incoherent aliasing artifacts (pseudo-noise) produced by overlapped coil sensitivities. In other words, the correlation among coil sensitivities limits the resulting intercoil incoherence and at least K samples per coil are required to recover the overall K sparse coefficients. This lower bound can be useful when deciding to use PI alone or CS & PI. If the compression ratio is higher than the maximum acceleration of PI alone, CS & PI would likely perform better.

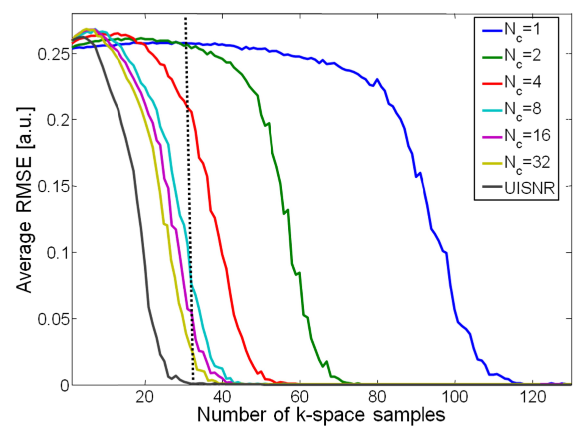

Fig. 2: Average RMSE for noise-free 1D simulations for planar arrays with different number of coils (N_c) and the ultimate-SNR array (UISNR). The dotted line denotes image sparsity (K=32).

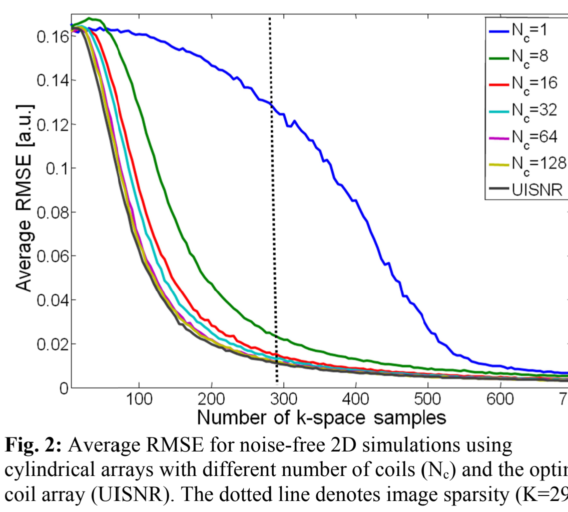

Fig. 2: Average RMSE for noise-free 2D simulations using cylindrical arrays with different number of coils (N_c) and the optimal coil array (UISNR). The dotted line denotes image sparsity (K=290).

REFERENCES: [1] Lustig M et al. ISMRM 2009; 379. [2] Liang D et al. Magn Reson Med 009;62 (6):1574-84. [3] Otazo R et al. Magn Reson Med. 2010;64(3):767-76. [4] Lustig M et al. Magn Reson Med. 2007; 58:1182-1195. [5] Tsaig Y et al. Signal Process. 2006; 86:533–548. [6] Ohliger M et al. Magn Reson Med 2003;50(5):1018-30. [7] Wiesinger F et al. Magn Reson Med 2004; 52(2):376-390. [8] Lattanzi R et al. ISMRM 2008; 1074. [9] Candès E et al. IEEE Trans Inf Theory 2006; 52:489–509. **GRANT:** NIH R01-EB000447