Linear Sweeps generate Extreme Echoes: explanation, generalization

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Introduction It has been observed (1) that the modulation of the phase of the refocusing pulses in an echo train according to a quadratic in the form $\varphi(i) = 2\pi(n/d)i^2$, where *i* is the pulse index, *n* and *d* integers, generates strong refocusing at the end of a period *d* if it is even or after two periods if it is odd. The figure 1 depicts an example with *n* =4 and *d*=21, for nutation of the refocusing pulses ranging from 180° down to 75°, with a step of 15°. A strong refocusing (Noted XE, for 'Extreme Echo' (2)) is observed at echo indices 42,84,..., and that even for 75° pulse nutation. This phenomenon is mathematically demonstrated in (3), but we present here a more physical explanation which also leads to some generalization.

Theory The figure 2 depicts the suite of rotations from echo to echo applied by each refocusing pulse once the receiver phase has been adapted to transform a quadratic phase modulation in a linear precession sweep (3,4). The rotation is along the indicated axis, with a length proportional to the applied rotation angle. This is for a proton with a given natural precession offset $0 < \omega T < 2\pi$, T being half the echo space (4) . Here we choose ωT such that the first rotation r_1 is in the position indicated with a rotation angle which is then π. We are interested by the cycle rotation exercised by one period of pulses on this proton, namely $R = r_{21} \cdots r_1$. Combining a suite of rotations is generally very painful but the striking result is that one has here $R = r_1$. It suffices to note that, at the middle of the sequence, one has $r_{11}r_{12} = 1$ so that one can eliminate these two factors. Iterating the same peel off ($r_{10}r_{13} = 1, \cdots r_2 r_{21} = 1$) one finds the announced result. But the angle of rotation of a sequence is not changed when permuting circularly the rotations. Hence a proton whose natural offset is different from the previous one by $2\pi\, n/d$, and thus sees a sequence of rotations $R' = r_1 r_{21} \cdots r_2$, will have a different cycle rotation axis, but still will have the same cycle rotation angle, π. More generally all the 21 such possible natural offsets have the same cycle rotation π . Moreover another collection of protons interleaved with the first can be obtained (with r_1 now at the extreme position close to r_9). Hence there are 42 equidistant natural precessions having a cycle rotation of π, for a cycle composed of 21 echoes, or 42 units of time, T. It is then easy to accept, by simple continuity, that the rotation angle does not deviate too much from π for any natural precession offset ωT (The demonstration and the necessary condition are given in (3)). But if the cycle rotation angle is almost uniformly π, it does not mean that a refocusing is created after one cycle because the cycle rotation axis itself is dispersed. But after two such cycles a quasi-perfect refocusing is indeed obtained. For a cycle with an even period, an equivalent approach shows that one cycle rotation is uniformly 2π, and generates an extreme echo after one period.

Figure 1

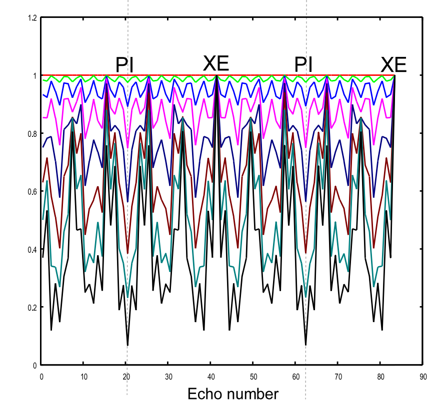


Figure 2

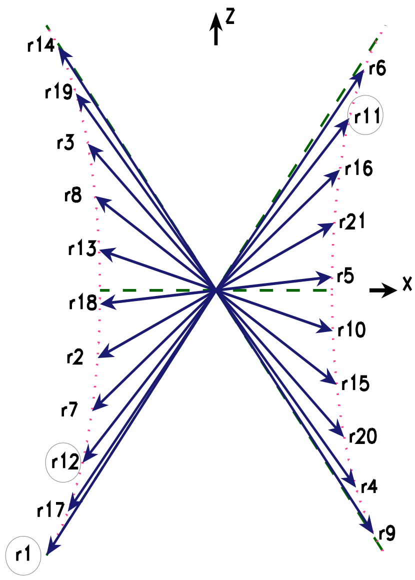
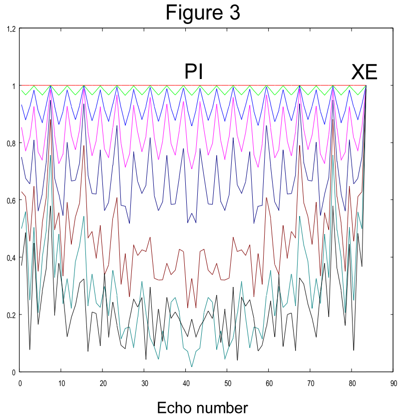


Figure 3



Generalization The genesis of extreme echoes thus relies on the symmetry of the echo to echo kernel rotation, followed by an equalization of the rotation angle performed by the linear sweep. This can be generalized to more complicate kernel presenting the same kind of symmetry of the rotation axis. Indeed if a suite of rotation is symmetrical in time (symmetrical phase modulation means anti symmetrical precession offset as above) the resulting signed axis of rotation is anti-symmetrical (5,6). An example is given by the XY modulation, lasting four echoes, that we swept 'by block' with a period of 21 blocks, generating an 82nd extreme echo, as shown in figure 3.

References(1) Murdoch,J.B.,SMR Scientific Meeting Proc. 1994, p.1145. (2) Murdoch, J.B, ISMRM Scientific Meeeting Proc. 2003; p. 2002. (3)Le Roux,P.,J. Magn. Res. 155, p.278, 2002. (4) LeRoux,P. et al, J. Magn. Res. 211, p.121, 2011. (5) Ngo,J.T, et al, J. Magn. Res. 74, p.122, 1987. (6) Shaka,A.J., et al, J. Magn. Res. 71, p.495, 1987.