GENERAL CLOSED-FORM EXPRESSIONS FOR DKI PARAMETERS AND THEIR APPLICATION TO FAST AND ROBUST DKI COMPUTATION BASED ON OUTLIER REMOVAL

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INTRODUCTION AND PURPOSE

Non-Gaussianity quantification of water diffusion through diffusional kurtosis imaging (DKI) [1] attracts clinical researches of its application to differentiation and classification between normal and abnormal tissues [2]. One of the important problems in DKI for clinical application is potentially its computational cost. Computation of diffusion kurtosis K with diffusion coefficient D (and also signal value for zero b-value; S_0) is performed through a solution of least square problem, and generally a numerical method via Levenberg-Marquardt (LM) algorithm [3] is used. Though simple closed-form solutions of K and D are presented for a special case of three b-values [4], it is desirable to use data set with more b-values for more reliable results. Another issue is computation robustness for clinical data processing. That is, a fast and robust computation technique feasible for clinical data is expected. In this short paper, we first show that it is possible to obtain closed-form expressions of K, D, and S_0 also for cases of more than three b-values. Then, we propose a robust DKI computation algorithm based on outlier sample removal. Validation studies were performed in the viewpoint of computation time in comparison with LM method, and on the basis of reduction of computation error.

THEORIES AND METHODS

General closed-form expressions: The least square fitting in DKI can be rewritten as a minimization problem of an objective function J below.

$$J = \sum_{\alpha=1}^{N} \left\{ \frac{1}{6} {b_\alpha}^2 D^2 K - b_\alpha D - \ln\left(\frac{S_\alpha}{S_0}\right) \right\}^2$$

Note that we have N kinds of b-values b_α (α=1…N) and corresponding signal values $S_\alpha = S(b_\alpha)$ measured in diffusion weighted imaging. For minimizing J with regard to K, D, and S_0, the conditions on partial derivatives at the stationary points can be used, that is $\partial J/\partial K = \partial J/\partial D = \partial J/\partial S_0 = 0$. Then these equations simply lead us to closed-form expressions of K, D, and S_0 as (omitting the proof for that Hessian of J is positive-definite),

$$D = \frac{(\Lambda_{10}\Lambda_{40} - \Lambda_{20}\Lambda_{30})\Lambda_{01} - (\Lambda_{40} - {\Lambda_{20}}^2)\Lambda_{11} - (\Lambda_{10}\Lambda_{20} - \Lambda_{30})\Lambda_{21}}{2\Lambda_{10}\Lambda_{20}\Lambda_{30} + \Lambda_{20}\Lambda_{40} - {\Lambda_{10}}^2\Lambda_{40} - {\Lambda_{20}}^3 - {\Lambda_{30}}^2},\; S_0 = exp\frac{(\Lambda_{10}\Lambda_{40} - \Lambda_{20}\Lambda_{30})D + \Lambda_{40}\Lambda_{01} - \Lambda_{20}\Lambda_{21}}{\Lambda_{40} - {\Lambda_{20}}^2},\text{ and } K = 6\frac{\Lambda_{30}D^{-1} + (\Lambda_{21} - \Lambda_{20} log S_0)D^{-2}}{\Lambda_{40}},$$

where $\Lambda_{ij} = \left\{\sum_{\alpha=1}^{N} {b_\alpha}^i \cdot (\log S_\alpha)^j\right\}/N$, such as $\Lambda_{40} = \left(\sum_{\alpha=1}^{N} {b_\alpha}^4\right)/N$ and $\Lambda_{21} = \left(\sum_{\alpha=1}^{N} {b_\alpha}^2 \cdot \log S_\alpha\right)/N$, for the instances.

Simultaneously, we can obtain the minimum value of the objective function; J_{min} as a closed-form expression consisting of Λ_{ij} and N as following.

$$J_{min} = N\left[\Lambda_{02} + \frac{2\Lambda_{20}\Lambda_{01}\Lambda_{21} - \Lambda_{40}{\Lambda_{01}}^2 - {\Lambda_{21}}^2}{\Lambda_{40} - {\Lambda_{20}}^2} - \frac{\left\{(\Lambda_{10}\Lambda_{40} - \Lambda_{20}\Lambda_{30})\Lambda_{01} - (\Lambda_{40} - {\Lambda_{20}}^2)\Lambda_{11} - (\Lambda_{10}\Lambda_{20} - \Lambda_{30})\Lambda_{21}\right\}^2}{(\Lambda_{40} - {\Lambda_{20}}^2)(2\Lambda_{10}\Lambda_{20}\Lambda_{30} + \Lambda_{20}\Lambda_{40} - {\Lambda_{10}}^2\Lambda_{40} - {\Lambda_{20}}^3 - {\Lambda_{30}}^2)}\right]$$

Because J is a summation of square errors at sample pairs of b-value and signal (b_α, S_α), J/N expresses the mean square error (MSE) per sample pair. In addition to the expressions including S_0 above, we can obtain the expressions of D and K in a similar way for the case we trust measured S_0 values.

DKI computation with outlier removal: In DKI images obtained from clinical data, voxels of extraordinary values due to computation error often appear as salt-and-pepper noise. Those values of D and K are often lower than the general lower bounds such as D>0.0 and K>-2.0 on physical bases. In addition, it was reported that K values in brain are around 1.0 [1], which can lead us to a practical range; 0<K<2.0. When we have certain amount of sample pairs (b_α, S_α), it is effective to remove outliers to reduce fitting error J_{min}. The RANSAC algorithm [5] is based on an idea of random sample removal to search better fitting. Because we have very limited number of samples, we can examine all combinations to find the least MSE instead of random combination. For instance, we only have to try ${}_NC_1=N$ times when single sample pair is removed. Because of the closed-form expressions, this algorithm is feasible enough for clinical data and standard PCs.

RESULTS AND DISCUSSIONS

Materials: Six brain DWI datasets of healthy volunteers were obtained by a clinical 3T MR scanner (Philips Achiva) with the acquisition parameters; TR/TE=3000/80 msec., 128x128 matrix, 6~32 MPG directions, seven b-values of 0, 500, 1000, 1500, 1500, 2000, and 2500 sec/mm^2. After removal of the bias due to Rician noise [6], DKI computation was performed with 4 methods from combinations of with/without S_0 estimation (wS0 or woS0) and with/without single outlier removal (wOR or woOR).

Experiments 1 (computation time): Based on the closed form expressions, we achieved over 30 times faster computation speed than the LM-based method. Because values of Λ_{ij} are independent of DWI signals when j=0, most part of the closed-form expressions can be commonly precomputed, which bring us further efficient DKI computation.

Experiments 2 (computation error ratio): For the DWI sets, we counted computation error voxels with values out of the range of (D>0, 0<K<2) in the presegmented brain mask. As shown in Fig.1, our algorithm with S_0 estimation and outlier removal achieved the lowest error ratio, especially in the K value computation. We observed that most of the removed samples were mainly from higher b-values. This is natural because higher b-values yield signals with lower signal-to-noise ratio in DWI. These results clearly show the feasibility and the effectiveness of our method applied in clinical DKI.

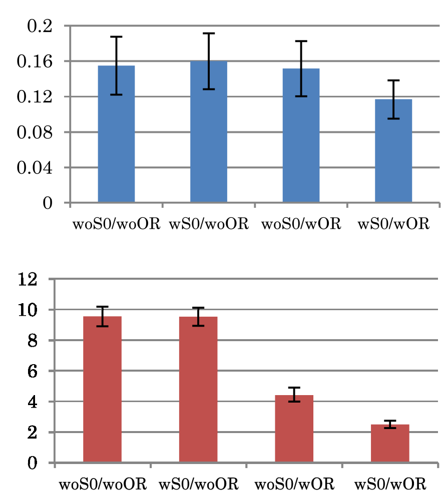

Fig.1 Error ratio (%) for D (upper) and K (lower) computation. Mean of all data sets are displayed.

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