Reliability of bi-exponential parameter estimation

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Introduction:

Bi-exponential parameter estimation with curve fitting is commonly used technique in many areas, and especially popular in diffusion MRI. Since the curve fitting process is an application of non-linear minimization, convergence to unique global minimum is not guaranteed. Existence of multiple local minima is often suggested, but exact shape of MSE (mean squared error) distribution in the parameter space is not known. In this work, this distribution was actually calculated and visualized, and it was shown that there exist a wide area with MSE close to the global minimum. By selecting a new parameter set which does not change over this area, it is possible to estimate parameters that define the system more reliably.

Methods:

A test data set was generated by below equation, with known parameters.

$$S_i = PD\,[f\exp(-b_i ADC_1) + (1-f)\exp(-b_i ADC_2)] \qquad [1]$$

Each parameter, ADC1, ADC2, f and PD, was changed over a predetermined range, and MSE corresponding the parameter set was plotted. To visualize the result, the minimum MSE was plotted over a combination of 2 parameters, for example, ADC1 and ADC2. Because this exhaustive search is a time consuming process, GPU was used to shorten the calculation time.

Results and discussion:

Results are shown in fig 1. White indicates larger error. The parameters used for this particular set is: ADC1 = 1.5, ADC2 = 0.5 $(\times 10^{-3} mm^2/s)$, f = 0.3, PD = 1.0, and the computation time was approximately 10 minutes with CPU (Intel Xeon), and 3 seconds with GPU (NVIDIA GeForce8800GT). There are small MSE area along curved lines, meaning parameters on these lines generate similar bi-exponential curves. From these plots, it is suggested that parameter estimation by curve fitting is not reliable. Fig 1 (left) is ADC1 vs. ADC2 plot. The area with minimum MSE forms a hyperbolic curve. Parameters on this curved line satisfies below:

$$(ADC_1 - D)(D - ADC_2) = d^2 \qquad [2]$$

where D is single exponential estimate of ADC, and d is a new parameter related to standard deviation. These parameters, D and d, (and PD) are reliably estimated by curve fitting. The graphical meaning of these parameters is shown on fig 1. The main reason of popularity of bi-exponential curve fitting is that it can represent biological data very well (ref 1). However, it is also pointed out that parameter estimation with this curve fitting is sometimes extremely difficult, because different parameter set can produce very similar curves. This is particularly true for small range of b-value, like 0 - 1000 (s/mm^2), which is often used for tumor diagnosis in the abdominal organs. Also, for the tumor diagnosis, it is necessary to interpret the measured data in terms of underlying tissue structure, which is also difficult with bi-exponential curve fitting.

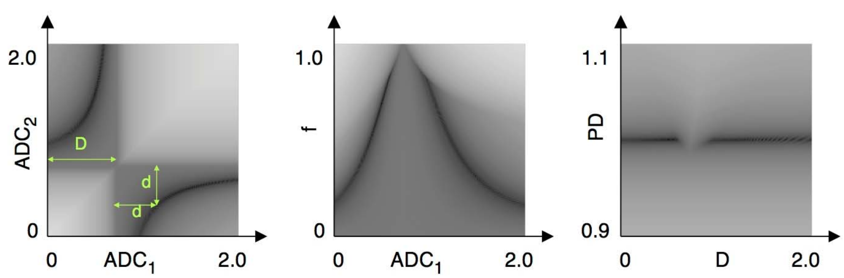

Fig. 1

It can be shown that these parameters, D and d, are roughly equal to ADC and □ in statistical model of diffusion (ref 2). In this model, highly complex system like tumor tissue is modeled as many "compartments", with distinctive ADC's. The mean and the standard deviation of this ADC distribution can be estimated using ordinary multi-b diffusion measurement. Using this new set of parameters, D and d, it is possible to estimate the parameters more reliably, and the parameter interpretation becomes more straightforward.

References:

1. Mulkern RV et al, MRI 27:1151-1162 (2009).

2. Yablonskiy AD et al, MRM 50:664-559 (2003).