

# The Parallel Kalman Filter: an efficient tool to deal with real-time non central $\chi$ noise correction

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## Introduction

The Parallel Kalman Filter (PKF) [1] is an incremental solver, thus very suitable for real-time (RT) purpose, like the Kalman Filter (KF). Moreover, the PKF prevails upon the KF, as it is adapted to non-Gaussian noise filtering, whereas the KF corrects Gaussian noise only. Here, we propose to employ it to correct diffusion-weighted (DW) data from Rician and non central  $\chi$  (nc- $\chi$ ) noise (the Rician noise being a particular case of the nc- $\chi$  noise), in a compatible way with the RT DW Magnetic Resonance Imaging (MRI) environment introduced by [2]. A RT correction method already exists, relying on a Linear Minimum Mean Square Error Estimator (LMMSE) derived for Rician noise [3] or extended to nc- $\chi$  noise [4], and whose corrected signal is then injected in a KF, embedded with a feedback loop [5]. This previous method assumes that the LMMSE leaves a Gaussian noise in the corrected signal. This hypothesis is precisely the major flaw of the technique because it is impossible to verify that the resulting noise is really Gaussian. Furthermore, it is highly sensitive to any error on the LMMSE estimation. To overcome this problem, we propose this new RT noise correction method based on the PKF, which directly embeds a non-Gaussian noise distribution through its Gaussian mixture representation.

## Material & Methods

**Noise correction** – The PKF purpose is to simplify the nc- $\chi$  noise correction problem in a Gaussian one, using the idea that the distribution of a signal corrupted by nc- $\chi$  noise can be approximated by a Gaussian mixture. To this aim: 1) the LMMSE is applied to the measured DW signal  $M(\mathbf{v}, \mathbf{o})$ , at position  $\mathbf{v}$  for the orientation  $\mathbf{o}$ , in order to give an estimate of the noise-free signal  $\hat{S}(\mathbf{v}, \mathbf{o})$ , which with an estimation of the noise standard deviation  $\hat{\sigma}$  (using the same method as in [5]) completely defines the nc- $\chi$  distribution of  $M(\mathbf{v}, \mathbf{o})$ . 2) This distribution can be decomposed in a Gaussian mixture using a Levenberg-Marquardt algorithm. 3) This mixture of  $f$  Gaussian distributions is injected in a PKF, which estimates, in RT, coefficients characterizing the noise-corrected signal in the modified Spherical Harmonics (SH) basis employed in the analytical Q-ball model [6] (fig. 1). The PKF (fig. 2) takes into account each Gaussian noise through linear systems working in parallel, which deliver a collapsed density being a Bayesian *a posteriori* estimate of the Gaussian mixture. This collapsed density has only one Gaussian term and can be injected in a Kalman-like filter, which gives the optimal corrected coefficients  $\hat{\mathbf{C}}_v^{\text{DWI}}$ . These coefficients, refined at each new iteration  $\mathbf{o}$ , are re-injected through a feedback loop in the LMMSE. The loop calculates a weight  $w(\mathbf{v}, \mathbf{v}')$  that constrains the LMMSE spatially and structurally for better accuracy, as in [5].

We compared this new RT method (PKF method) with the one proposed by [5] on a simulated DW field at  $b=4500\text{s/mm}^2$ , artificially corrupted by nc- $\chi$  noise for a number of channels  $n=4$  as in [5], and on real DW data with the following parameters:  $5 \times 5 \times 5$  neighborhood, Laplace-Beltrami regularization factor  $\lambda=0.006$ , maximum SH order  $N=8$ . To take into account effects from correlations between the  $n$  channels, an effective number of channels  $n_{\text{eff}}$  was calculated, as advised by [7], but in an empirical way by choosing it when yielding the highest SNR after correction.  $n_{\text{eff}}=2.0$  and  $2.6$  respectively for the PKF method and [5].

**Acquisitions** – Single shot DW Spin Echo EPI acquisitions were performed on a 3T Tim Trio (Siemens, Erlangen) with following parameters: 60 diffusion directions,  $b=0/4500\text{s/mm}^2$ , matrix  $128 \times 128$ , 60 slices,  $\text{FOV}=19.8\text{cm}$ , resolution  $1.7 \times 1.7 \times 1.7 \text{ mm}^3$ ,  $T_E/T_R=116\text{ms}/14\text{s}$ , GRAPPA factor=2, partial Fourier 5/8,  $\text{RBW}=1628\text{Hz/pixel}$  and an architecture of  $4 \times 3$  coupled channels, yielding  $n=4$ .

## Results & Discussion

Fig. 3 shows, on the simulated data, a decreased  $\epsilon$  after both methods for a large panel of noise levels. Nevertheless, the crossing information in the ODF is correctly retrieved only with the PKF method. Fig. 4 depicts, on the real data, the contrast improvement gained with both methods, with higher smoothing level for [5]. The PKF provides more accuracy towards the anatomical structures in the RGB map which is confirmed on the ODFs: whereas the method by [5] neglects the raw angular information, the PKF keeps it to produce a sharper ODF with decreased bias.

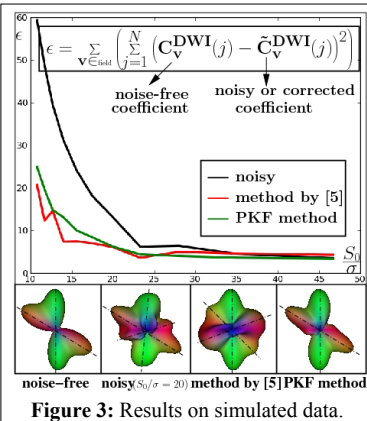


Figure 3: Results on simulated data.

al., IEEE TMI 2008; 27. [4]Brion *et al.*, in 14<sup>th</sup> MICCAI. [5]Brion *et al.*, in 28<sup>th</sup> ESMRMB. [6]Descoteaux *et al.*, MRM; 28. [7]Aja-Fernández *et al.*, MRM 2011.

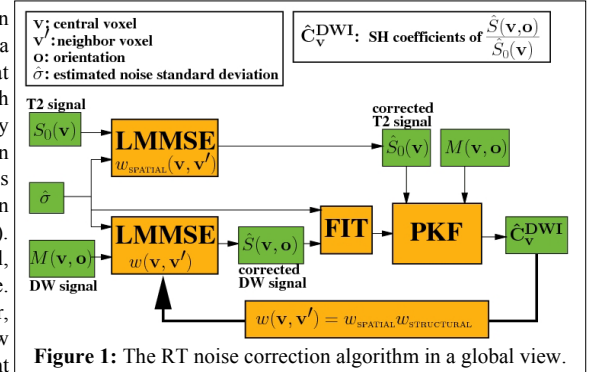


Figure 1: The RT noise correction algorithm in a global view.

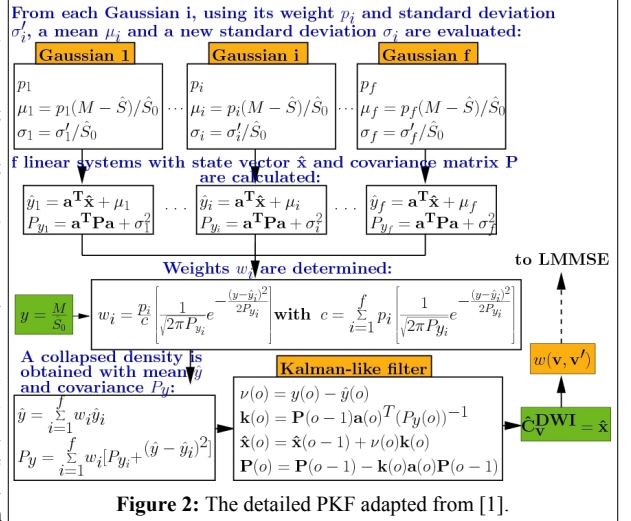


Figure 2: The detailed PKF adapted from [1].

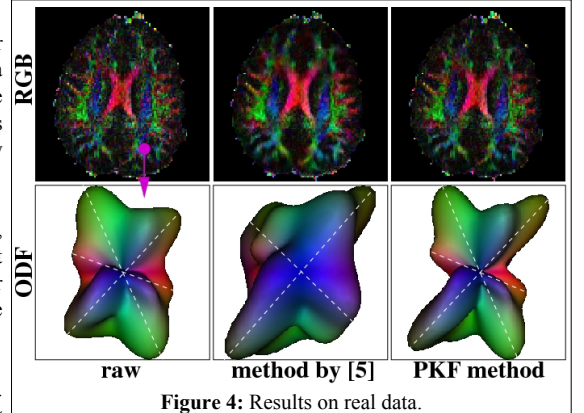


Figure 4: Results on real data.