COMPUTED DIFFUSION WEIGHTED IMAGING UNDER RICIAN NOISE DISTRIBUTION

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Introduction: Body diffusion imaging to detect cancer [1] is widely applied. A technique named computed diffusion imaging (cDWI), allowing to provide high b-value (b) equivalent DWI images using relatively low b-value DWI images, was proposed as a useful technique for improving contrast between tumor and the background tissue in body diffusion imaging, compared to actually measured DWI (mDWI) with high b-value [2-4]. cDWI was clinically evaluated and its signal-to-noise ratio (SNR) model based on Gaussian noise distribution was also introduced [2]. However, it is not clear the dependency on b-value or tissue apparent diffusion coefficient (ADC) to optimize tissue contrast on cDWI under Rician noise[5]. Here we assessed the relationship among b-values, SNR of original data, and ADC values for obtaining cDWI with higher accuracy and precision based on simulation and volunteer study.

Methods:

Theory: The cDWI signal at $b=b_c$ was obtained by $S_c=S_0\exp[-(b_c-b_0)ADC]$ after calculating ADC by $ADC=\ln[-S_m/S_0]/(b_m-b_0)$ using 2 points mDWI signals S_0, and S_m, respectively with n_0 signal averages at $b=b_0$ and n_m signal averages at $b=b_m$. Assuming noise is Gaussian distributed, the standard deviation (SD) of noise on the mDWI is independent of b_m; while noise SD of cDWI signal, σ_c depends on both b_c and b_m and modified as:

$$\sigma_c = \sigma_0 \left(S_m/S_0\right)^{(b_c/b_m)}\left[\left\{1-(b_c/b_m)^2\right\}+(n_0/n_m)(b_c/b_m)^2\left(S_m/S_0\right)^{-2}\right]^{\frac{1}{2}} \qquad (1)$$

where σ_0 is the noise SD on the signal S_0. In contrast, it is known that the signal intensity under Rician noise is dominantly biased when SNR< 2 [5].

***Monte-Carlo simulation*:** Assuming Rician distribution of noise on magnitude DWI signals, noise SD and the mean of mDWI and cDWI signals were assessed after 1000 times each trial as a parameter of b-value and ADC. Here ADC was varied 0.5-2.0 x10^{-3} mm^2/s considering the range in living subject. σ_c/σ_m vs. b_c when b_0=0 and b_m=1000, and the mean±SD of cDWI signal vs. b_m when b_0=0 and b_c=2000 were assessed to optimize b_m for cDWI.

***Phantom and Volunteer Study*:** A phantom including acetone, water, dimethyl sulfoxide (DMSO), and celery and volunteer of pelvic axial images were assessed. Imaging was performed on 1.5-T MR imager of Toshiba Excelart/Titan^{TM} with single-shot SE-EPI of b_0=0 and b_m=1000. TE was common for phantom but minimized depending on b-value for volunteer and all other parameters were kept constant among DWIs. cDWI of b_c=2000 s/mm^2 was calculated then measured the mean and SD in the ROI on each subject .

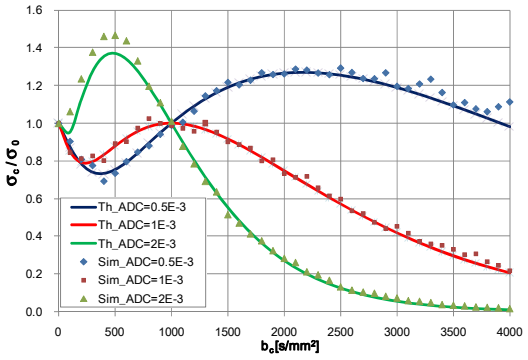

Fig. 1 Noise SD ratio to σ_0 vs. b_c for cDWI under theoretical Gaussian noise (line) and simulated Rician noise (dot) when $n_0=n_m$, S_0/σ_0= 20, and b_m=1000 s/mm^2. All curves pass through (0,1) and (b_m,$(n_0/n_m)^{0.5}$) because cDWI and mDWI become equal at these points. Note that the noise-SDs on cDWI are less than on the mDWI when $b_c>b_m$ for $ADC>1/b_m$ (1E-3 and 2E-3).

Results and Discussion: The noise SD vs. b_c for both Gaussian and Rician provided almost the same results if $S_m/\sigma_0>3$ was satisfied and cDWI noises became less than the mDWI noises when $b_c>b_m$ for $ADC>1/b_m$(**Fig. 1**). The b-value of 1000 s/mm^2 was regarded as optimal to minimize errors in ADC and cDWI signal for the ADC range between 0.5 and 2.0 (**Table.1**), which corresponds to the inverse of the average of ADCs. As shown also in phantom (**Fig. 2**) and volunteer study (**Fig. 3**), noise bias artifacts became greater for mDWI signals of high ADC (=2.0x10^{-3} mm^2/s), which resulted in the reduction of CNR between low and high ADC signals; in contrast, cDWI signals generated from b_m=1000 s/mm^2 were further reduced than the mDWI signals for greater b-values in high ADC subjects. The cDWI can provide better SNR than the mDWI at $b>b_m$ as long as the condition of $SNR>3$ at $b=b_m$ is satisfied by reducing background tissue signals below noise bias, which was problematic for high ADC tissues with high b-values on standard mDWI. In conclusion, cDWI technique with optimal parameters was confirmed to provide high CNR DWI imaging especially for higher ADC and shorter T2 tissues in body diffusion under Rician noise with lower hardware loading .

References: [1]Takahara *et al*., Radiat Med 2004; 22(4):275-282,[2]Blackledge et al., ISMRM 2009:pp4005, [3]Blackledge et al., ISMRM 2010:pp4707, [4]Blackledge et al., ISMRM 2011:pp2979, [5]Gudbjartsson et al. MRM 34:910-914(1995).

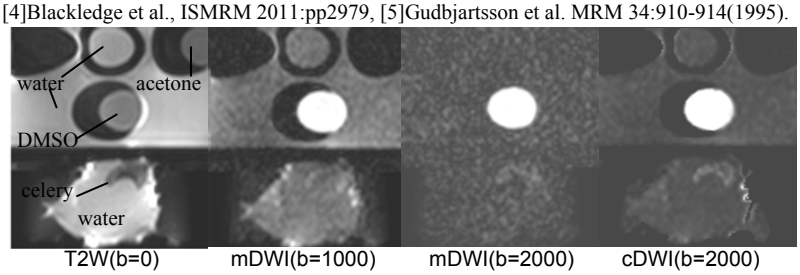

Fig. 2 Phantom DWI images. TE of 90ms was all common. Note that cDWI showed better SNR especially for celery than the mDWI image at the same b-value of 2000.

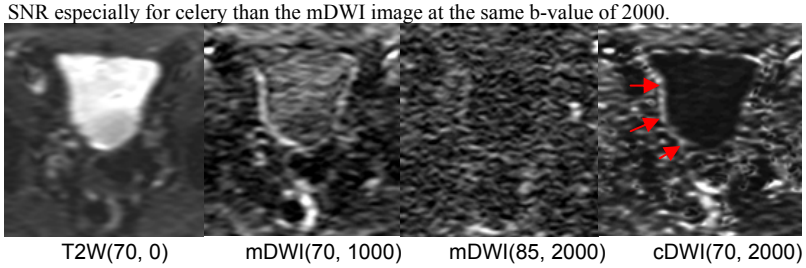

Fig. 3 Normal volunteer axial pelvic DWI images. Note that cDWI showed better contrast than the mDWI of b=2000 especially for the wall of urinary bladder (arrows); the wall-to-inside CNR for mDWI and cDWI were respectively 3.1 and 21.7 where the improvement of CNR on cDWI contributed both effects of the TE reduction and the cDWI σ_c. Parameters in ( ) denote (TE[ms],b[s/mm^2]).

Table. 1 Simulated ideal-related errors of ADC and cDWI signals of b_c=2000 vs. b_m for three different ADC assuming Rician noise where S_0/σ_0= 20. The errors of both the mean and SD were enhanced notably for greater ADC at greater b_m. The average errors for three ADCs were minimized at b_m=1000 s/mm^2.

ADC[mm^2/s]	b_m[s/mm^2]	500	1000	1500	2000
0.5E-3	ADC[%]	1.4 ± 31.8	0.6 ± 19.5	0.0 ± 16.3	1.1 ± 14.4
	cDWI(2000)[%]	2.8 ± 30.3	0.6 ± 17.5	0.9 ± 14.7	0.1 ± 13.4
1E-3	ADC[%]	0.3 ± 19.5	0.4 ± 15.4	2.4 ± 18.4	3.7 ± 23.3
	cDWI(2000)[%]	6.2 ± 39.0	3.0 ± 29.0	1.0 ± 32.4	-1.0 ± 37.0
2E-3	ADC[%]	1.4 ± 14.8	4.7 ± 25.6	8.4 ± 36.4	-11.6 ± 27.5
	cDWI(2000)[%]	10.3 ± 62.6	12.8 ± 73.9	30.8 ± 113.5	131.0 ± 168.2
average	ADC[%]	1.0 ± 22.1	1.9 ± 20.2	3.6 ± 23.7	-2.3 ± 21.7
	cDWI(2000)[%]	6.4 ± 44.0	5.5 ± 40.1	10.9 ± 53.6	43.4 ± 72.9

Table. 2 Mean and noise SD for mDWI and cDWI of ROI in the phantom (Fig. 2) where the noise-bias was around 20. The mean value as well as noise SD in acetone and water for cDWI signals were dramatically reduced below the noise bias while the mDWI could not.

subject	DMSO	water	acetone	celery
ADCx10^{-3}[mm^2/s]	0.83	2.24	3.53	1.49
mDWI(b=2000)	284.3±18.3	22.8±13.4	19.6±10.2	33.3±10.8
cDWI(b=2000)	260.6±18.1	14.8±3.5	0.6±0.6	22.9±5.1