Denoising Diffusion-Weighted MR Images Using Low Rank Structure and Edge Constraints

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Introduction: Significant effort has been made to address the noise issue in diffusion imaging. Previously proposed denoising approaches work either on the complex data or solely on the magnitude images. While complex domain methods [1-2] have the advantages in performance and theoretical characterization, methods working on magnitude images have also received great attention due to the absence of phase artifacts and convenient access to magnitude data [3-7]. However, these existing methods have not taken full advantage of the intrinsic properties and prior information about the diffusion-weighted (DW) image sequence. In this work, we propose a novel approach to jointly denoise a sequence of magnitude DW images. The proposed penalized maximum likelihood (PML) formulation combines Rician signal modeling [3-7], low rank modeling exploiting the correlation within the DW image sequence [8] and prior edge information from high SNR images [9]. The proposed method is evaluated using experimental diffusion tensor imaging (DTI) data, and is shown to provide superior performance in recovering image features, anisotropy and orientation information of diffusion tensors originally corrupted by noise.

Theory: Given a sequence of noisy magnitude DW images $\mathbf{Y} = [\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_Q]$, where each vector $\mathbf{y}_q$ stands for one image frame, the goal in denoising is to estimate the noise-free image sequence $\mathbf{X} = [\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_Q]$. We address this problem using a PML estimation formulation as follows:

$$\mathbf{X}^* = \arg\max_{\mathbf{X}} \mathrm{p}(\mathbf{X} \mid \mathbf{Y}) = \arg\min_{\mathbf{X}} -\log \mathrm{p}(\mathbf{Y} \mid \mathbf{X}) - \log \mathrm{p}(\mathbf{X}), \quad (1)$$

where $\mathrm{p}(\mathbf{Y}|\mathbf{X})$ is the likelihood for the noisy data, $\mathrm{p}(\mathbf{X})$ is the image prior reflecting the intrinsic properties and side information about the image sequence. Using independent Rician modeling of the signal distribution [3], we can express $\mathrm{p}(\mathbf{Y}|\mathbf{X})$ as

$$\mathrm{p}(\mathbf{Y} \mid \mathbf{X}) = \prod_{q=1}^{Q}\prod_{m=1}^{M} \frac{y_{mq}}{\sigma^2} \exp\left(-\frac{y_{mq}^2 + x_{mq}^2}{2\sigma^2}\right) I_0\left(\frac{y_{mq}x_{mq}}{\sigma^2}\right),$$

where $I_0(.)$ is the modified Bessel function of the first kind with order zero, σ^2 is referred to as the noise variance, and m and q are the indices for the image voxel and image frame respectively. The proposed image model contains two important, complementary components: 1) exploiting the correlated diffusion weighted behavior within the image sequence, and 2) utilizing the edge information from one or multiple reference images. Specifically, we model the image sequence $\mathbf{X}$ as $\mathbf{X}=\mathbf{U}\mathbf{V}$, where $\mathbf{U}\in \mathrm{R}^{M\mathrm{x}L}$ and $\mathbf{V}\in \mathrm{R}^{L\mathrm{x}Q}$ are low rank matrices with $L<Q<<M$. We also incorporate prior edge information in the form of $\mathrm{p}(\mathbf{X}) \propto \exp\left(-\lambda \sum_{q=1}^{Q} \left\|\mathbf{W}\mathbf{D}\mathbf{x}_q\right\|_2^2\right)$, where $\mathbf{x}_q$ is the q^{th} noise-free frame, $\mathbf{D}$ is a finite difference operator, $\mathbf{W}$ is a weighting matrix, and λ is a constant which determines the smoothing strength. Integrating the Rician signal model and the image model into the PML formulation in (1), the proposed joint optimization problem is given as follows

$$\hat{\mathbf{U}}, \hat{\mathbf{V}} = \arg\min_{\mathbf{U},\mathbf{V}} \sum_{q=1}^{Q}\sum_{m=1}^{M} \frac{y_{mq}^2 + (\mathbf{U}\mathbf{V})_{mq}^2}{2\sigma^2} - \log I_0\left(\frac{y_{mq}(\mathbf{U}\mathbf{V})_{mq}}{\sigma^2}\right) + \lambda \sum_{q=1}^{Q} \left\|\mathbf{W}\mathbf{D}(\mathbf{U}\mathbf{v}_q)\right\|_2^2 .$$

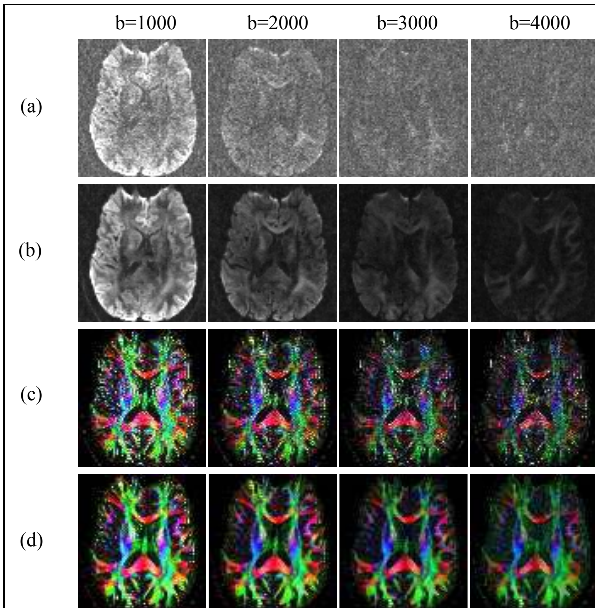

Fig.1: Illustration of denoising performance of the proposed method. The four rows correspond to (a) Representative image frames from noisy data; (b) Corresponding frames after denoising by the proposed method; (c) FA maps with color-coded direction information from the primary eigenvector of the estimated diffusion tensors from noisy images; (d) Corresponding estimate from the denoised images. The results for different b-values are shown in different columns.

We solve this problem iteratively using a BFGS Quasi-Newton algorithm to estimate $\mathbf{U}$ and $\mathbf{V}$ in an alternating fashion [10]. $\mathbf{U}$ and $\mathbf{V}$ were initialized by applying SVD to the noisy image sequence. The estimate of σ^2 was obtained as half of the mean square intensity in the background region [5].

Methods: We evaluated the proposed method using different sets of experimental DTI data. One of them was acquired with a 3T Siemens scanner using a spin-echo EPI sequence (32 channels, 128x128 matrix size, 256x256mm^2 FOV, and 35 slices). DW image sequences were acquired with multiple b-values (b=1000, 2000, 3000, 4000s/mm^2), and 30 diffusion encoding directions were acquired for each b-value. Partial Fourier reconstruction [11] was applied to each coil. The proposed method was then applied to denoise the reconstructed images and sum-of-squares was used to combine images from all coils for diffusion tensor estimation.

Results: Figure 1 illustrates the performance of the proposed method on this data set. As can be seen, the proposed method not only significantly reduces the noise in the DW images and reveals features concealed by noise, but also recovers the diffusion tensor information accurately. Consistent improvement can be observed for data acquired at different b-values. We have also observed the benefit of the proposed method taking into account the low rank property of the signal and the prior information, in various simulation settings and with different experimental data sets. The results are not shown here due to space limitation.

Conclusion: We proposed a novel formulation for jointly denoising a sequence of magnitude DW images. The proposed method combines Rician signal modeling with a novel image model based on low rank structure and prior edge constraints. The effectiveness of the proposed method has been evaluated using experimental DTI data. We expect the proposed method to be useful for achieving higher measurement precision and/or reducing data acquisition time for diffusion MRI.

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