The Rician bias in diffusion MRI: a technical overview

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Purpose: Diffusion magnetic resonance imaging (dMRI) is currently the only method for the in vivo and non-invasive quantification of the diffusion of water molecules in biological tissue. During the last decade, several diffusion models were proposed to obtain quantitative diffusion measures, and/or fiber reconstructions to gain information on the structural and organizational features of healthy and impaired biological tissues, the brain white matter (WM) in particular. Since most of the diffusion models require many highly diffusion-weighted MR images (DWIs), dMRI commonly suffers from low signal-to-noise ratio (SNR). It is clear that the precision of the diffusion model parameter estimators depends on the SNR, however the estimator's accuracy will also be affected, e.g., if the Rice distribution of magnitude MR data is not accounted for. We would like to give a technical overview of –on the one hand - the effect of the so-called Rician bias in dMRI studies and - on the other hand – a selection of techniques, which were proposed to reduce/remove the Rician bias. The pros and cons of the different techniques will be discussed and compared.

Outline of content:

Rician distributed diffusion weighted data: Both the real and imaginary components of the complex-values MR signal are independently normal distributed. As motion and flow will introduce phase shifts, the magnitude-reconstructed signals are most commonly preferred in quantitative dMRI studies. The magnitude of bivariate normal random variables is however Rice distributed. Therefore, the probability distribution function of the DW magnitude signal, *m*, is [1]:

$$p(m|\nu,\sigma) = \frac{m}{\sigma^2}\exp\left(-\frac{m^2+\nu^2}{2\sigma^2}\right) I_0\left(\frac{m\nu}{\sigma^2}\right)$$

with ν the noise-free signal magnitude, σ the Gaussian noise standard deviation and I_0 the zeroth-order modified Bessel function. The conditional expectation is given by:

$$\mu = E(m|\nu,\sigma) = \sigma\sqrt{\frac{\pi}{2}} L_{1/2}\left(-\frac{\nu^2}{2\sigma^2}\right)$$

with $L_{1/2}$ a Laguerre polynomial.

Rician bias on diffusion measures

At high SNR, the Rice distribution can well be approximated by a normal distribution. However, at low SNR, the Rice distribution significantly deviates from a Gaussian distribution. As a result, its expectation value, μ, exceeds the underlying noise free signal intensity. Hence, applying parameter estimation methods that assume Gaussian distributed data to low SNR DW data will result in biased diffusion measures. Since the SNR of DW data drops with increasing b-value, the signal amplitude will increasingly be overestimated. That *b*-value dependent overestimation of the DW signal amplitude causes the diffusion model parameters to suffer from a *Rician bias*. For Diffusion Tensor Imaging (DTI), the *Rician bias* corresponds to a direction-dependent underestimation of the apparent diffusion coefficient (ADC), causing underestimation of the mean diffusivity and the fractional anisotropy (FA) [2]. Overestimation of the FA due to eigenvalue repulsion has also been reported [3]. For higher order diffusion models, such as Diffusion Kurtosis Imaging (DKI), the Rician signal bias will be a source of artificial non-Gaussian behavior. The non-Gaussian appearing of Gaussian diffusion will, for example, result in an overestimation of kurtosis parameters when the DKI model is used [4].

Rician bias reduction in diffusion model fitting

(A) Maximum Likelihood estimator (MLE) [5]: The diffusion model parameter vector was estimated from the *N* independent DW signals, ***S***, with a ML estimator in each voxel by substituting the observed values for the stochastic variables and maximizing over the parameters:

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \sum_{n=1}^{N} \ln p(S_n|\hat{S}_n(\boldsymbol{\theta}),\sigma_n)$$

Vector $\hat{\boldsymbol{S}}(\boldsymbol{\theta})$ contains the *N* DW signals, $\hat{S}_n(\theta)$, reconstructed with the preferred diffusion model. The MLE has in theory favorable properties: asymptotical consistency, normality and efficiency. In practice, this MLE might suffer from a significant bias due to motion and eddy current corrections – applied prior to model fitting – which alter the DW data distribution.

(B) Conditional least squares (CLS): One can minimize the sum of squared deviations about the conditional expectation of a Rice distributed DW signal. Given a set of DW measurements, ***S***, we estimate the diffusion model parameters, $\boldsymbol{\theta}$, by minimizing the conditional sum of squares:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \left\| \boldsymbol{S} - \boldsymbol{\sigma}\sqrt{\frac{\pi}{2}} L_{1/2}\left(-\frac{\hat{\boldsymbol{S}}(\theta)}{2\boldsymbol{\sigma}^2}\right) \right\|_2^2$$

The assumption of normally distributed error terms renders maximum likelihood estimation equivalent to the minimization of a sum of squares. The method of conditional least squares (CLS) enjoys consistency and asymptotic normality, under some mild regularity conditions [6]. The diffusion model parameters are estimated by solving the equation using Levenberg-Marquardt optimization. Weights should be applied to cope with the signal dependency of the variance of a Rice distribution. Prior knowledge of the noise level is recommended because of the lack of a unique solution in some cases (e.g. isotropic diffusion). If DW data is corrected for motion and eddy current distortions prior to model fitting, the CLS estimator remains asymptotically unbiased in high SNR or homogeneous regions.

(C) Bias reducing nonlinear least squares (brNLS): At moderate SNR, the conditional expectation of a Rice distribution can be approximated [1]: $\mu \approx \sqrt{\nu^2+\sigma^2}$ resulting in following estimator:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \left\| \boldsymbol{S} - \sqrt{\hat{\boldsymbol{S}}(\boldsymbol{\theta}) - \boldsymbol{\sigma}^2} \right\|_2^2.$$

This approximation of the CLS is a fast way to reduce the Rician bias, but the approach still suffers from a bias when using high b-valued DW data.

Rician noise level estimation in diffusion weighted data

Noise estimation is a challenging, though important task, because the accuracy and precision of the parameter estimation will depend on the prior knowledge of the noise level. Most of the existing noise estimation methods often fail in dMRI because of the spatial dependency of the noise, low signal-to-noise ratio (SNR) and the restricted spatial resolution. We will give a brief overview of noise level estimators developed for dMRI and we will compare them with our own (unpublished) approach based on a 3D wavelet transformation of the DW data [7] and an iterative correction scheme [8].

Summary: The Rician bias becomes increasingly important in dMRI because of the growing use of high *b*-values. In this work, we will firstly give a technical overview of the effect of the Rician bias in dMRI, with a focus on DTI, DKI and constraint spherical deconvolution (CSD). Secondly, we will compare several estimators, which account to certain degree for the Rice distribution of DW data. Finally, we will discuss the importance of prior noise level estimation and how this estimation is challenging in dMRI. Several approaches for noise level estimation will be compared.

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