Rotationally Invariant Gradient Schemes for Diffusion MRI

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Introduction

Minimizing the error propagation that a diffusion MRI (dMRI) gradient scheme introduces is an important task in the design of robust and un-biased experiments. Previous studies define the optimal single-shell HARDI scheme with respect to various parameters that include: 1) the angular distance between neighboring samples, minimized using an electrostatic optimization [Jones99] 2) the condition number, which estimates the effect of noise [Skare00] and 3) rotational invariance [Skare00, Batchelor03], so the scheme produces rotationally unbiased estimates. Among the previously proposed schemes, the electrostatic optimization has been shown to produce the most balanced schemes [Hasan01, Batchelor03]. In addition, it was shown that the icosahedron set provides optimized schemes, albeit for specific number of samples [Hasan01, Batchelor03]. Most HARDI acquisitions are usually composed of a single b-value shell providing angular sampling of the diffusion profile. But the newer methods require, in addition, a radial sampling in order to observe phenomena such as restriction and hindrance [Assaf04]. These methods use a number of b-value shells, usually reaching to high values (b>1500 s/mm^2). We propose two schemes for the construction of rotationally invariant multiple-shells. The first is a dual frame method that optimizes the rotation invariance of any set of samples. The second uses a subset of the icosahedral set that can intuitively be used for nested rotationally invariant schemes with pre-defined number of samples.

Theory

Dual frame scheme: The directions obtained from the electrostatic optimization [Jones99] defines an optimally spread sampling, yet not necessarily rotationally invariant [Batchelor03]. The directions X obtained from the optimization define a frame (an overcomplete basis). Here we modify this frame X by defining its dual frame. The dual basis is calculated as $\tilde{X} = (XX')^{-1}X$. In the case of a frame that is not rotational invariant, the dual frame defines a second scheme, which has the opposite deviation from the first scheme. Therefore an invariant scheme can be found between those two extremes. Here we estimate the rotational invariant scheme by using the half way dual transform $(XX')^{-0.5}$ iteratively: $X_t = (X_{t-1}X_{t-1}')^{-0.5}X_{t-1}$ where $X_0 = X$. The rotational invariance of the electrostatic scheme, and the dual frame scheme (Fig 1) show that the optimization of the electrostatic approach is not rotationally invariant (blue curve is sometimes below the optimum value of 1) whereas our new approach (green) always yields the optimal value 1. The condition number and the normalized electrostatic energy of the proposed schemes remain very similar to the electrostatic schemes.

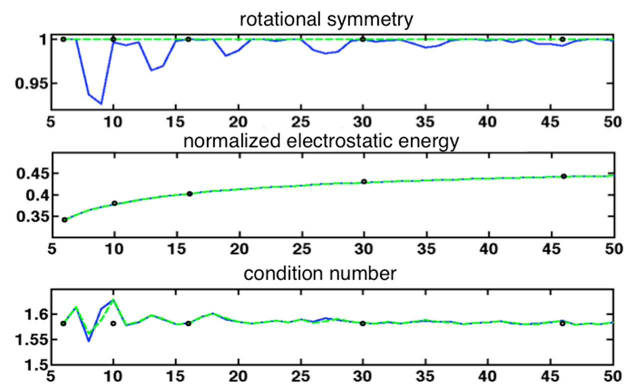

Fig 1. Top: Rotational symmetry of electrostatically optimized schemes (blue), and the rotationally corrected dual frame scheme (dashed-green). Note that the green curve is at the optimum value of 1 for all sets of directions (6-50 directions). Middle: normalized electrostatic energy. Note that the energy for the corrected scheme is very similar to the uncorrected one. Bottom: The condition numbers (of the diffusion design matrix) for the two schemes are also very similar.

Geometric multi-shell scheme: Figure 1 also demonstrates that the icosahedral set [Hasan01, Batchelor03] (black circles) always has optimal rotation invariance and a constant condition number. The icosahedron (Fig 2, top left) and the dodecahedron (Fig 2, top middle) have a special dual relationship: they have faces and vertices interchanged. Thus each center of a face in the icosahedron is a vertex in the dodecahedron. Thus their vertices are perfectly interleaved and are complementary as a sampling pattern. Another complementary sampling pattern can be introduced by selecting two samples on each of the ridges of the icosahedron, dividing the ridge into thirds. This pattern corresponds to the truncated icosahedron (Fig 2, top right). The number of independent diffusion directions from these geometries is half of the number of vertices: 6 for the icosahedron, 10 for the dodecahedron, and 30 for the truncated icosahedron. Since the sampling patterns are complementary, they can be combined in a single rotationally invariants shell (Fig 2, bottom), thus we can straightforwardly construct shells with: 6, 10, 16, 30, 36, 40, and 46 samples. For example, nested complementary (interleaved) sampling with 5 shells can be achieved with shells that are spaced linearly in q (which is quadratic in b-value, b≈q^{2}Δ) using the scheme with b-value (number of directions): 0(1), 120(6), 480(10), 1080(16), 1920(30), 3000(46). This particular sampling takes less than 20 minutes on a GE, 3T scanner.

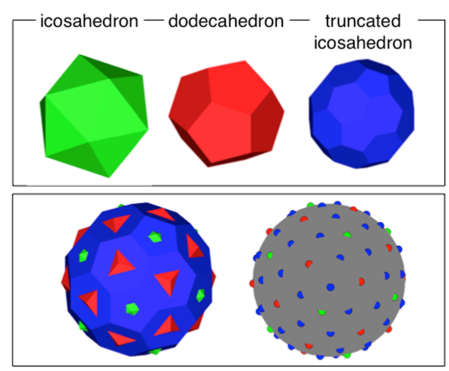

Fig 2. Top: Visualization of the icosahedron, the dodecahedron, and the truncated icosahedron. Bottom: Combined visualization (left), the corners represent the gradient orientations (right).

Discussion

We present new schemes for dMRI acquisitions that improve the rotational invariance of the existing schemes without jeopardizing the condition number. Our results suggest that the dual frame scheme we propose would make a better basis than the electrostatic optimized frame for multi shell schemes via methods that combine shells such as [Caruyer11]. Rotational invariance reduces the bias of the sampling, which is an important feature for reconstruction methods. Also, combining the multiple-shell approach with compressed sensing is likely to reduce the number of samples required for a given image quality, in which case we believe that the geometric multi-shell scheme proposed here will be sufficient for many applications.

References: [Jones99] Jones et al., MRM 42, 1999. [Skare00] Skare et al., JMR 147, 2000. [Batchelor03] Batchelor et al., MRM 49, 2003. [Hasan01] Hasan et al., JMRI 13, 2001. [Assaf04] Assaf et al., MRM 52, 2004. [Caruyer11] Caruyer et al., MICCAI 2011.