Optimal kinetic PASL design and CBF estimation with low SNR and Rician noise

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Introduction: Kinetic ASL imaging can provide important information in the care of stroke and brain tumor patients, including such parameters as CBF, CBV, transit delay, and MTT. By designing optimal observation times (TI) in PASL, Xie et al achieved more accurate estimation of perfusion parameters [1]. Their result was based on the assumption that the noise could be modeled as Gaussian, because of high SNR from signal averaging. However, magnitude images actually have Rician noise and in rapid kinetic multi-TI ASL, the SNR may be too low to approximate Rician noise as Gaussian. Here, we present the optimal design of an ASL experiment for CBF estimation from a one-compartment perfusion model and compare accuracy using both least square (LS) and L1-norm estimators with different SNR values and noise models.

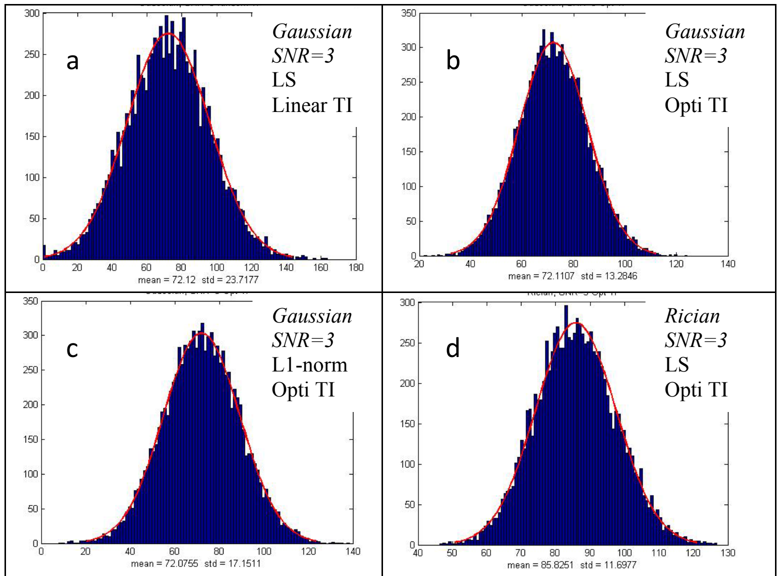

Figure 1. Efficient CBF estimation by LS and bias in Rician noise
(x-axis: CBF(ml/100g/min), y-axis: statistical count)
Optimal TI, Gaussian noise, LS estimation (b) gives the most accurate and unbiased estimation. Under the same conditions, Linearly-spaced TI (a) and L1 norm estimation (c) results have bigger variance. Rician noise model (d) results in significantly biased estimation. The true CBF is 72ml/100g/min.

Theory: In estimation theory, the estimation of a parameter from noisy data is subject to the Cramér–Rao lower bound:

$$E[(\hat{x}-x)^2 \mid x] \geq \frac{1}{E[(\frac{\partial \ln p(y \mid x)}{\partial x})^2]}$$

In choosing TIs, we try to maximize the Fisher matrix (the denominator) so as to minimize the variance of estimated parameters. This lower bound can be approached by maximum likelihood estimation (MLE) when the data set is large enough, regardless of the noise model. In the case of additive white Gaussian noise, LS estimation equates to MLE and therefore yields an efficient estimator, which should be better than an L1-norm estimator. However, when the SNR is low, the Rician noise can no longer be accurately approximated by a Gaussian model. Thus, LS is no longer a good estimator and results in biased estimation. The classic single-compartment PASL model [2] is used in estimation.

$$\Delta M(t,f,\Delta t) = \begin{cases} 0 & 0 < \mathrm{t} < \Delta \mathrm{t} \\ 2M_0 f(t-\Delta t)\alpha e^{-t/T1b} q(t) & \Delta \mathrm{t} < \mathrm{t} < \Delta \mathrm{t} + \tau \\ 2M_0 f\tau\alpha e^{-t/T1b} q(t) & \Delta \mathrm{t} + \tau < \mathrm{t} \end{cases} \quad q(t) = \begin{cases} \dfrac{e^{kt}(e^{-k\Delta t} - e^{-kt})}{k(t-\Delta t)} & \Delta \mathrm{t} < \mathrm{t} < \Delta \mathrm{t} + \tau \\ \dfrac{e^{kt}(e^{-k\Delta t} - e^{-k(\tau+\Delta t)})}{k\tau} & \Delta \mathrm{t} + \tau < \mathrm{t} \end{cases} \quad k = \frac{1}{T_{1b}} + \frac{1}{T_1} - \frac{f}{\lambda}$$

Methods: All simulations were performed using MATLAB 2011b. Tissue T1=1300ms, blood T_{1b}=1600ms, labeling efficiency α=0.9, brain-blood partition coefficient λ=0.9, duration of labeling blood τ=700ms, bolus arrival time Δt=700ms, CBF=72ml/100g/min was used to generate a perfusion signal, then different noise was added. Δt and CBF are assumed unknown and to be estimated from this signal. First, 10 TIs were calculated for optimal CBF estimation with a Gaussian noise model. Then, the CBF was estimated using both LS and L1-norms both with these optimal TIs and with linearly spaced TIs (100~3000ms). Each case was repeated 10000 times with different SNR levels and noise models to evaluate the estimation performance.

		Gaussian Noise		Rician Noise	
		L1 norm	LS	L1 norm	LS
SNR = 3	Opti-TI	72.08±17.15	72.11±13.28	84.07±15.77	85.82±11.70
	Linear TI	72.10±29.54	72.12±23.72	100.25±25.72	102.13±20.03
SNR = 10	Opti-TI	71.95±5.18	72.04±4.07	73.12±5.11	73.15±3.99
	Linear TI	71.98±8.92	71.93±7.26	74.98±8.76	74.88±7.07

Results and Conclusion: As shown in Fig. 1, in the case of low SNR Gaussian noise, LS is more accurate than L1-norm and performs better when using optimal TIs. In the case of low SNR Rician noise, which is significantly different from the Gaussian assumption, the estimator results in biased estimation. More results from the table show that when the SNR is high (SNR=10), Rician noise is similar to Gaussian noise and both estimators yield approximately unbiased estimation. In theory, optimal TIs can reduce the lower bound by 75%, which results in a lower standard deviation, as shown in the table. In this work, we have initial results showing that optimizing the observation TIs can achieve significantly improved estimation performance, regardless of noise model. Furthermore, low SNR and thus Rician noise could result in biased CBF estimation; an optimal unbiased estimator needs to be developed.

References: 1. Xie et al., *MRM*, 59:826-834 (2008). 2. Buxton et al., *MRM*, 40:283-396(1998).

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