Calculation of Electrical Properties from B1+ Maps - A Comparison of Methods

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Introduction: Extracting electrical properties (relative permittivity and conductivity) from B1+ maps is a promising, non-invasive method that has applications in local SAR estimation[1], RF hyperthermia treatment planning and diagnosis of tissue malignancy[2]. The method relies on the spatial variation of B1+, calculating the permittivity and conductivity at a given voxel using B1+ in the "neighborhood" of the voxel. In this work, we compare the two primary calculation methods, Laplacian based [3-5] and Integral based [6] under the constraint that both methods use a constant number of B1+ samples to estimate electrical properties.

$$\varepsilon_r(\mathbf{x}) = \frac{-1}{\omega^2\mu\varepsilon_0} Re\left\{\frac{\nabla^2 B_1^+(\mathbf{x})}{B_1^+(\mathbf{x})}\right\} \quad \text{-- (1)}$$

$$\sigma(\mathbf{x}) = \frac{1}{\omega\mu} Im\left\{\frac{\nabla^2 B_1^+(\mathbf{x})}{B_1^+(\mathbf{x})}\right\} \quad \text{-- (2)}$$

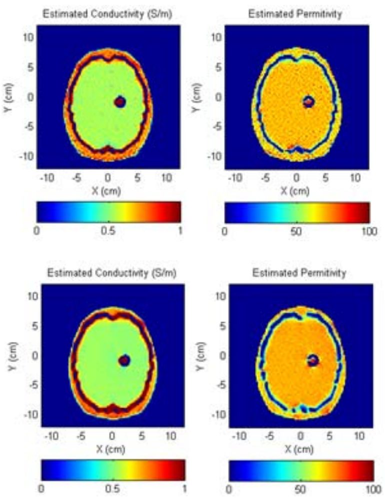

Figure 1 Conductivity and permittivity images: Laplacian (top) , Integral (bot)

$$\frac{\partial^2}{\partial x^2} B_1^+(i,j,k): \frac{B_1^+(i+2,j,k) - 2B_1^+(i,j,k) + B_1^+(i-2,j,k)}{4\Delta x^2} \quad \text{-- (3)}$$

Methods: The Laplacian based equations to calculate permittivity and conductivity from B1+ are given in equations (1) and (2). We constraint the number of B1+ samples to be 5×5×5 such that the permittivity and conductivity at location (i,j,k) are calculated using 5×5×5 B1+ data points, centered at (i,j,k). Calculating the Laplacian using difference equations, (i.e. equation (3) for x-direction) satisfies this constraint.

$$\varepsilon_r(\mathbf{x}) = \frac{-1}{\omega^2\mu\varepsilon_0} Re\left\{\frac{\oint_S \nabla B_1^+(\mathbf{x})\cdot d\mathbf{s}}{\int_V B_1^+(\mathbf{x})dv}\right\} \quad \text{-- (4)}$$

$$\sigma(\mathbf{x}) = \frac{1}{\omega\mu} Im\left\{\frac{\oint_S \nabla B_1^+(\mathbf{x})\cdot d\mathbf{s}}{\int_V B_1^+(\mathbf{x})dv}\right\} \quad \text{-- (5)}$$

The Integral based equations (eqns. (4) and (5)) require integration of the gradient of B1+ over the closed surface S covering volume V and normalization by B1+ integrated in the same volume V. Select V to be 2×2×2 voxels or equivalently 3×3×3 samples (note: this is different from the sample size constraint). Integrating the gradient of B1+ over the surface of 2x2x2 voxels using difference equations (i.e. equation (6) for gradient along x-direction at face ($i+1,j,k$)) requires 5×5×5 B1+ samples, meeting the constraint.

$$\frac{\partial}{\partial x} B_1^+(i+1,j,k): \frac{B_1^+(i+2,j,k) - B_1^+(i,j,k)}{2\Delta x} \quad \text{-- (6)}$$

Simulated B1+ maps were generated of a human head model in a 16-rung birdcage coil and permittivity and conductivity were estimated using both methods, under the constraint of 5×5×5 samples. Non-physical outcomes (e.g. permittivity outside 0 – 100 range, and conductivity outside 0 – 2 S/m range) generated near electrical properties boundaries were discarded. The results were compared with known electrical properties of the model. Next, zero mean, complex Gaussian noise (real and imaginary noise std. dev =0.053% relative to mean |B1+|) was added to the B1+ maps and the calculations were repeated.

Permittivity estimates			
Tissue	**Model**	**Laplacian**	**Integral**
Brain	**68**	**65.7**	**66.2**
Head outer	**64.7**	**59.8**	**55.9**
Tumor	**78**	**71.8**	**69.7**
Conductivity (S/m) estimates			
Brain	**0.54**	**0.58**	**0.58**
Head outer	**0.74**	**0.75**	**0.81**
Tumor	**1.6**	**1.59**	**1.59**

Table 1 Estimated electrical properties

Results: The permittivity and conductivity images generated from each method are shown in Fig. 1. The mean of estimated values, excluding the discarded values, are shown in Table 1, together with model parameters. The estimates were in good agreement with model parameters. The normalized root mean squared error (NRMSE) was less than 0.32 with Laplacian method (all tissues, permittivity and conductivity) and was less than 0.35 with Integral method. Figure 2 shows the permittivity and conductivity images generated from complex noise added B1+ maps. Although the image quality had decreased, the NRMSE was less than 0.44 (Laplacian), and 0.52 (Integral), excluding the discarded values.

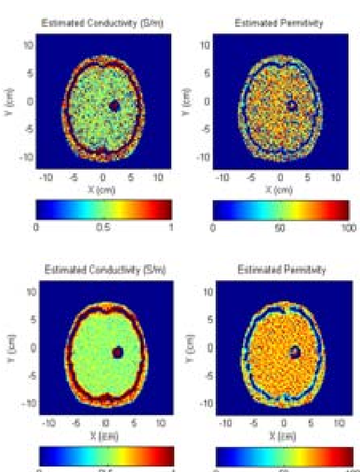

Figure 2 Electrical properties from noisy B1+, Laplacian (top), Integral (bot)

Discussion: The Laplacian calculation required next to nearest neighbor points, rather than nearest neighbor points. Therefore, the calculation required only three B1+ maps, rather than five B1+ maps, although the constraint was 5x5x5 samples. In practice, electrical properties can be estimated by acquiring B1+ maps only for the three required slices, skipping the intermediate slices. As the total time for electrical properties estimate is dominated by imaging time for B1+ mapping, Laplacian based method is advantageous for rapid electrical properties imaging. The integral based method requires more calculation steps as well as five B1+ maps. However, it is more robust to additive noise (Fig. 2) and is advantageous when B1+ maps have lower signal to noise ratio.

References: [1]Katscher, et al.,IEEE Trans Med Imaging,2009, [2]Van Lier, et al.,ISMRM,2011, [3]Wen,Proc. of SPIE,2003, [4]Bulumulla, et al.,ISMRM,2009, [5]Cloos, et al.,ISMRM,2009, [6]Voigt, et al.,Magn Reson Med,2011