

# THE TIP-ANGLE-DOUBLING METHOD AND ITS APPLICATIONS TO LARGE TIP ANGLE PULSE DESIGN

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**Introduction** Many clinical MRI sequences require large tip angle pulses. While RF design techniques to obtain small tip angle spatially selective pulses have been known since the first days of MRI [1], designing RF and gradients which obtain large tip angles is still a subject of current research [2,3]. In this work, we introduce the Tip Angle Doubling (TAD) principle, which makes possible to use small tip angle design methods to achieve large tip angles at no extra computation costs. Bloch equation simulations show how the method works in practice. The method allows fast large tip angle pulse design and is applicable to multi transmit systems with  $B_1^+$  inhomogeneities correction [4] and (local) SAR optimization [5].

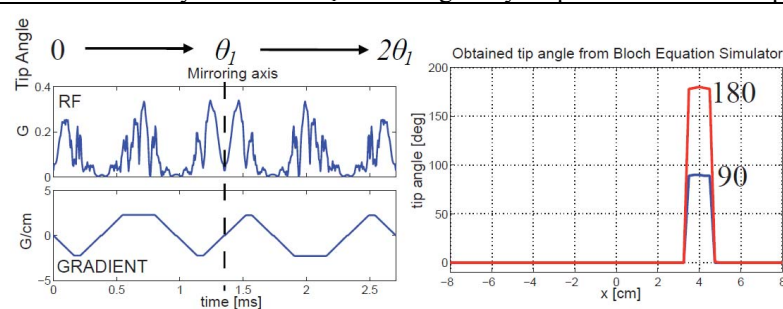
**Methods** Suppose a (time dependent) RF waveform,  $b(t)$ , and a gradient  $G(t) = (G_x(t), G_y(t), G_z(t))$  are applied to a spin system (denoted by the unit vector  $\mathbf{e}_0$ ) which is initially aligned along the main magnetic field  $B_0 = (0,0,B_0)$ , i.e.  $\mathbf{e}_0 = (0,0,1)$ . The RF and gradient have the discrete representation  $b_j = b(j\Delta)$  and  $G_j = G(j\Delta)$  where  $j=0,\dots,N-1$ ,  $\Delta$  is the scanner dwell time and  $N$  the number of sample points. The spins undergo a series of rotation which can be described by the product of the rotation matrices  $R_{\text{tot}} = R_{N-1}R_{N-2}\dots R_1R_0$  where  $R_j = R(b_j, G_j)$  is the rotation matrix given by the RF and gradients at the time point  $j\Delta$  with the axis of rotation given by the vector  $(\text{real}(b_j), \text{imag}(b_j), G_j \cdot r)$ . The tipped spin is then  $\mathbf{e}_1 = R_{\text{tot}} \mathbf{e}_0$ . The achieved tip angle is  $\theta_1 = \angle(\mathbf{e}_0, \mathbf{e}_1)$ . We define the *conjugated rotation matrix* given by  $\hat{R}_j = R(b_j, -G_j)$  (Note the minus sign for the gradient). Similarly, we introduce the *symmetrically conjugated total rotation matrix*  $\hat{R}_{\text{tot}} = \hat{R}_0 \hat{R}_1 \dots \hat{R}_{N-2} \hat{R}_{N-1}$ . Applying  $\hat{R}_{\text{tot}}$  to the tipped spin  $\mathbf{e}_1$ , we obtain a new vector  $\mathbf{e}_2 = \hat{R}_{\text{tot}} R_{\text{tot}} \mathbf{e}_0$ . It can be proven (the proof is omitted for lack of space) that for the obtained flip angle,  $\theta_2 = \angle(\mathbf{e}_0, \mathbf{e}_2)$ , it holds that  $\theta_2 = 2\theta_1$ . This fact can be exploited by designing an RF and gradient waveform for a small tip angle ( $[0^\circ, 90^\circ]$  range), then apply the mirrored RF and mirrored reversed gradient to obtain twice the tip angle, which will be in the  $[0^\circ, 180^\circ]$  range (Fig. 1). Note that the tip-angle-doubling (TAD) method works also for multi-dimensional parallel excitation RF with  $B_1^+$  inhomogeneity correction and (local) SAR optimization [5], since the SAR of the TAD pulse is exactly twice the SAR of the corresponding small tip angle pulse.

**Materials** As a first test, we construct a 1D RF/Gradient waveform to excite a block-shaped slab (4 cm off-center) with  $90^\circ$  tip angle according to [4]. The RF and Gradient are then assembled according to the TAD principle to obtain  $180^\circ$ . Bloch equation simulations are performed for the  $90^\circ$  pulses and for the corresponding  $180^\circ$  TAD pulses. As a second test, a  $180^\circ$  2D selective excitation profile is desired, see Fig. 3. A 2ch 7T birdcage head coil loaded with an oil spherical phantom is employed.  $B_1^+$  maps are measured with the AFI method [6] (see Fig. 2).

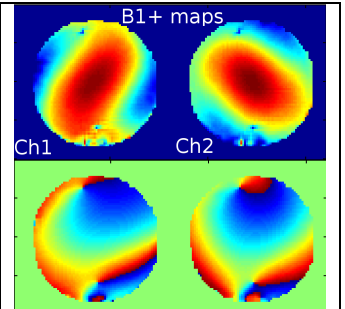
**Results** First test: see Fig. 1. Note that indeed, the tip angle profile of the TAD pulse is a scaled version of the simple pulse, with scaling factor of 2, achieving thus  $180^\circ$ . Second test: see Fig. 3. The magnetization profile is simulated also in case of  $\pm 50\text{Hz}$  and  $-50\text{Hz}$  off-resonance. Note the high accuracy of the magnetization profile obtained with the TAD pulse also in off-resonance regime.

**Conclusions** The TAD method makes possible to tackle large tip angle pulses design with the standard tools of small tip angle design. The method is applicable to multi channel systems with  $B_1^+$  inhomogeneity maps with local SAR optimization.

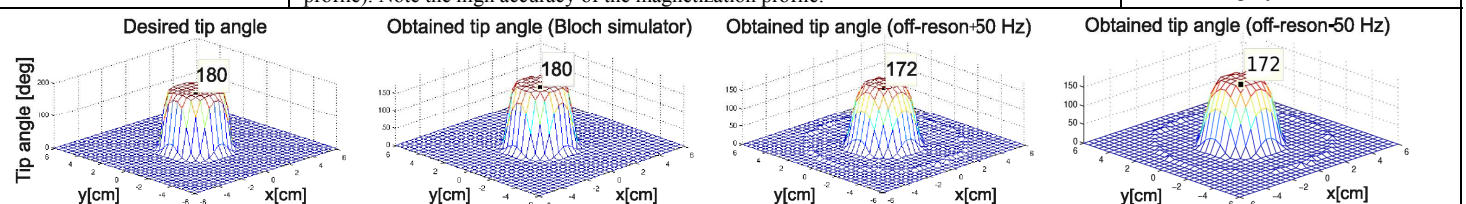
**References** [1] Pauly J et al JMR 81:43-56 (1989) [2] Xu D et al. MRM 59(3): 547-560 (2008) [3] Grissom WA et al. MRM 59(4): 779-787 (2008) [4] Katscher U et al. MRM 49:144-150 (2003) [5] Sbrizzi et al, MRM 2011 in press [6] Yarnykh VL et al. MRM 57:192 (2007)



**Figure 1** Left: The Tip angle principle applied to a slice selective pulse. Right: The obtained tip angle from the first part of the pulse (blue profile) and the whole TAD pulse (red profile). Note the high accuracy of the magnetization profile.



**Figure 2** The amplitude and phase maps of the 2ch birdcage headcoil employed for test 2.



**Figure 3** The desired and obtained magnetization profiles for the 2ch RF/G waveforms of the 2D pulse (test 2). Note the robustness of the pulse w.r.t.  $\pm 50\text{Hz}$  off-resonance.