# A Numerical Comparison of Quantitative Susceptibility Mapping Methods on Simulated Magnetic Field Maps

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**Introduction** Quantitative susceptibility mapping (QSM) is an emerging technique for quantification of MRI contrast agent. However, an ill-conditioned inverse problem leads to streaking artifacts as converting MRI phase data to susceptibility map. Although several studies have been proposed to eliminate the artifact in QSM, a quantitative comparison among these methods is still lacking. Therefore, the purpose of this study is to quantitatively compare the performance of those QSM methods by using numerical analysis and find the optimal conditions by comparing noise variation and contrast-to-noise ratio (CNR).

**Theory** In Fourier domain, the relationship of the susceptibility $\chi(k)$ and measured field $\delta(k)$ is as following:

$$\delta(k) = D(k)\cdot\chi(k) = \left(\frac{1}{3} - \frac{k_z^2}{k^2}\right)\cdot\chi(k)$$

where *D(k)* is the transfer function that is zero at conical surface and the inverse problem from magnetic field to susceptibility distribution is an ill-conditioned problem. In this study, a total of four QSM methods were compared. A linear method to avoid this divided-by-zero problem, referred to as threshold method [1], is to set a threshold value to establish a new transfer function *D'(k)*, where $D'(k) = D(k) \;\; as \;\; D(k) > t; \;\; D'(k) = t \;\; as \;\; D(k) \le t$. However, decreasing threshold value *t* commonly causes unpredictable noise amplification. To minimize this artifact, three alternative regularization methods using the iterative non-linear approach, Tikhonov regularization, MEDI [2] [3] and Total Variation (TV) [4], were used,

Tikhonov: $$\chi^* = \min\|\delta - F_D\chi\|_2 + \lambda\|\chi\|_2$$

MEDI: $$\chi^* = \min|\nabla\chi|_1 + \lambda\|\delta - F_D\chi\|_2$$

Total Variation (TV): $$\chi^* = \|\delta - F_D\chi\|_2 + \lambda|\nabla\chi|_1$$

$F_D = F^{-1}D(k)F$ that *F* is 3D fast Fourier transform, λ is a weighting coefficient and $\nabla$ is divergence operator.

**Material and methods** As shown in fig. (a), the simulated spherical susceptibility distribution was created in a matrix space of 256×256×32 voxels with the parameters of 1 ppm and radius of 10. The direction of $B_0$ was constrained to y-axis. Zero-mean white noise with two different values of standard deviation (SD) of 0.05 ppm and 0.01 ppm were added to the simulated field maps. The values of signal-to-noise ratio (SNR) of these magnetic field maps were approximate 50 and 10 as illustrated in fig. (b) and (c), respectively. To investigate the noise effect, four QSM algorithms were used for comparison in this study, including threshold, Tikhonov, MEDI and TV. The data analysis was performed in Matlab 2011a (The Mathworks Inc., MA).

**Results** The susceptibility maps derived from simulated magnetic field using threshold, Tikhonov, MEDI and TV methods with SNR values of 50 and 10 were shown in figures (d) and (e), respectively, suggeting that both Tikhonov and TV methods lead to smaller streaking artifact. Further, it also shows that TV could potentially eliminate noise propagation efficiently. As shown in table 1, according to Rose Model (CNR>5), it shows that the data acquired using TV method could provide higher sensitivity among these four methods.

**Conclusion** In this study, four QSM methods have been compared by using the numerical analysis. Although threshold method was fast and simple, but subjected to undetermined susceptibility values and dramatic streaking artifact. Those non-linear regularization methods could potentially reduce the noise propagation, but were time-consuming due to higher computational complexity. In summary, by quantitative comparison, our results suggested that TV regularization method could be a robust method in susceptibility mapping and potentially helpful to further applications.

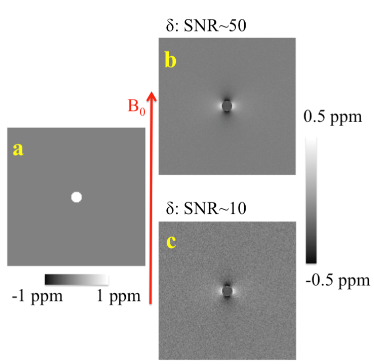


Fig. (a) Spherical susceptibility distribution. (b)(c) Simulated magnetic field map with SNR values of 50 and 10.

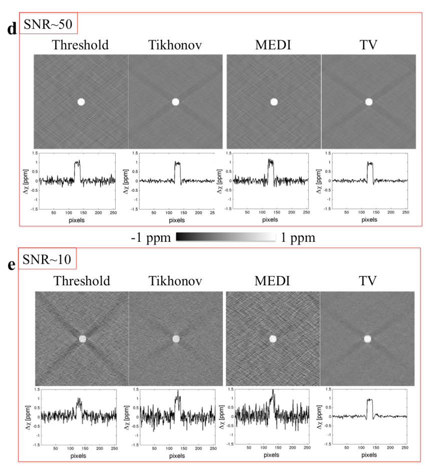


Fig. (d)(e) The susceptibility maps derived from simulated magnetic field using threshold, Tikhonov, MEDI and TV methods with SNR values of 50 and 10, respectively.

| | Method | Threshold | Tikhonov | MEDI | TV |
|---|---|---|---|---|---|
| SNR~50 | Noise SD (ppm) | 0.13 | 0.05 | 0.11 | 0.05 |
| | CNR | 7.60 | 17.67 | 9.05 | 19.09 |
| SNR~10 | Noise SD (ppm) | 0.19 | 0.27 | 0.27 | 0.12 |
| | CNR | 3.59 | 3.29 | 3.39 | 7.67 |

Table 1.Noise SD and CNR of susceptibility maps.

**Reference** [1] Shmueli et al. MRM (2009) [2] Liu et al. MRM (2011) [3] Zhu et al. UCLA CAM Report: 2008 [4] Dey et al. Mic. Res. and Tech. (2006)