Regularized Harmonic Estimation for Steady-State MR Elastography

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Introduction: In steady-state magnetic resonance elastography (MRE) [1,2], the delivery of precise quantitative information about tissue stiffness inherently depends on accurate estimation of the invoked harmonic phase signal. Typically, the phase contrast signals associated with each offset in the harmonic cycle are first independently estimated from the corresponding phase contrast data sets using some type of aggregate estimator (e.g., [3]). The resulting signal is then temporally Fourier transformed, and the first harmonic component is isolated. While straightforward, this approach does not explicitly account for noise present in the base signal, which inevitably propagates into the generated harmonic estimate. Moreover, this noise manifests in the harmonic signal according to a complex and signal-dependent distribution, which complicates retrospective denoising. Inspired by Funai et al.'s [4] work on robust field map estimation, we propose a statistical strategy for estimating the first harmonic signal directly from the raw, complex MRI data, and discuss the incorporation of signal prior models for prospective noise suppression (as opposed to retrospective noise removal).

Theory: The signal observed by the c^{th} coil element of a phase array receiver at time offset t during positive/negative motion encoding can be modeled as

$$g_{\pm}(x,c,t)=m_0(x,c,t)e^{\pm j\phi(x,t)}+n_{\pm}(x,c,t) \quad (1)$$

where m_0 is the (possibly time-varying) complex background signal, φ is the motion-induced phase signal, and $n_{\pm}(\cdot,\cdot,c)$ is complex Gaussian noise with variance σ_c^2 (assumed uncorrelated across channels). Regularized maximum likelihood (ML) estimation of m_0 and φ corresponds to

$$[\hat{m}_0,\hat{\phi}]=\arg\min_{m_0,\phi}\{\alpha\,\mathrm{P}(\phi)+\mathrm{L}(m_0,\phi)\} \quad (2)$$

where α is a mixing parameter, P is a penalty functional and the likelihood functional

$$\begin{aligned}\mathrm{L}(m_0,\phi)=&\sum_{c=0}^{C-1}\sum_{t=0}^{T-1}\sum_{x\in\Omega}\sigma_c^{-2}\left|m_0(x,c,t)e^{+j\phi(x,t)}-g_+(x,c,t)\right|^2\\&+\sum_{c=0}^{C-1}\sum_{t=0}^{T-1}\sum_{x\in\Omega}\sigma_c^{-2}\left|m_0(x,c,t)e^{-j\phi(x,t)}-g_-(x,c,t)\right|^2\end{aligned} \quad (3)$$

As the induced phase signal φ is, in theory, harmonic, it can be expressed as

$$\phi(x,t)=|\eta(x)|\cos(\angle\eta(x)+\angle\zeta(t))=\tfrac{1}{2}\left(\eta(x)\zeta(t)+\bar{\eta}(x)\bar{\zeta}(t)\right) \quad (4)$$

where η is the targeted (complex) harmonic component and $\zeta(t)=\exp(j2\pi t/T)$. As L is linear with respect to m_0, its closed-form minimizer can be embedded into (3), reducing it (after much simplification [4]) to

$$L(\eta)=\alpha\,\mathrm{P}(\eta)+\sum_{c=0}^{C-1}\sum_{t=0}^{T-1}\sum_{x\in\Omega}\sigma_c^{-2}\left|g_+(x,c,t)g_-(x,c,t)\right|\left(1-\cos\left(\eta(x)\zeta(t)+\bar{\eta}(x)\bar{\zeta}(t)-(\angle g_+(x,c,t)-\angle g_-(x,c,t))\right)\right) \quad (5)$$

Minimizers of (5) can be efficiently computed, e.g., via nonlinear conjugate gradient iteration or preconditioned gradient descent [4]; note, however, that solutions are not unique due to the possibility of phase wrapping (implicit to the cos term).

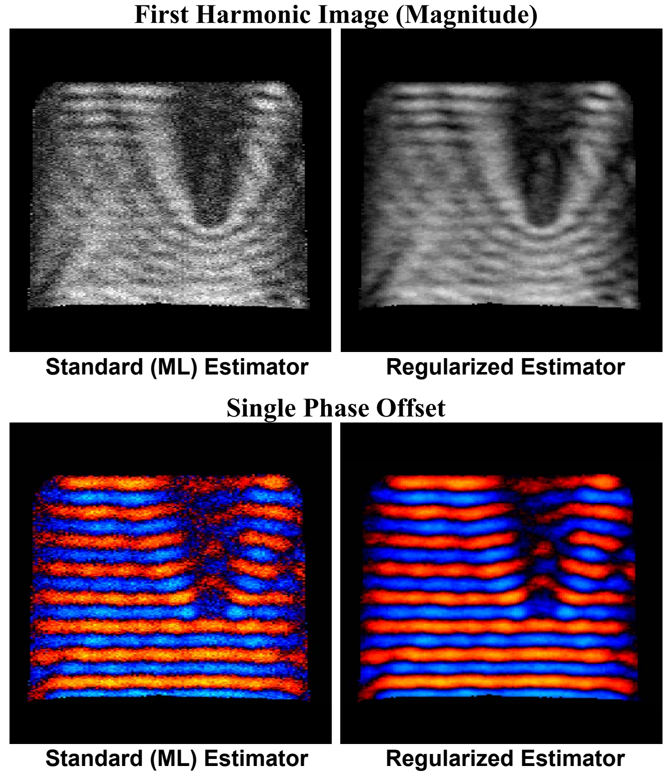

Fig 1) ML vs regularized harmonic estimation. The lower phase offset images were generated from the harmonic estimates.

Example: Figure 1 shows example first harmonic estimate images for an agar gel phantom with two stiffness inclusions, imaged at 1.5T with a single-channel receiver and using a standard 2D phase contrast MRE sequence (GRE, 8 phase offsets, 300Hz excitation) [1,2]. Both ML and regularized harmonic estimation results are depicted; ML estimation corresponded to α=0, whereas α=1e5 was manually selected for regularized estimation. Harmonic estimation was executed by 250 iterations of a diagonally preconditioned gradient descent [4] to minimize (5), with penalty functional, P(), defined as the L_2-norm of the signal transformed via 1^{st} order finite differences (along both x and y). A Matlab implementation of this code, on a 3.0GHz Pentium 4 machine with 4Gb memory required about 1 minute to execute. For both the directly estimated harmonic image and the phase offset image generated from it, observe that noise present in the ML images is largely absent in the regularized estimate images; however, there is no obvious structural loss in the regularized result (see, also, Fig. 2).

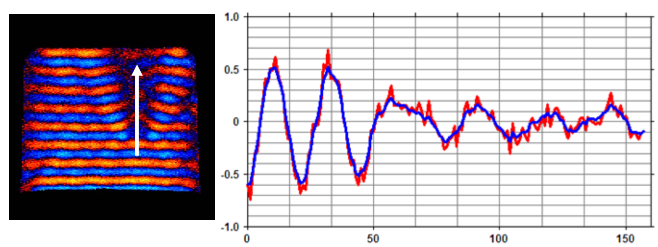

Fig 2) Phase offset image profiles (red=ML est., blue=reg. est.)

Discussion: We have proposed a statistically-motivated framework for estimating the first harmonic signal directly from the raw, complex MRI data, and demonstrated the potential benefit of incorporating regulatory terms for prospective noise suppression. Future directions include the investigation of alternative penalty models, such as sparsity-based regularization (e.g., using wave atoms [5]), direct incorporation of directional filtering strategies [6], and the generalization of the proposed models for direct first harmonic reconstruction from undersampled k-space data.

References: [1] R. Muthupillai et al., Science 269(5232):1854-1857, 1995; [2] A. Manduca et al., Med. Imag. Anal. 5(4):237-254, 2001; [3] M. Bernstein et al., Mag. Res. Med. 32(3):330-334, 1994; [4] A. Funai et al., IEEE Trans. Med. Imag. 27(1):1484-1494, 2008; [5] L. Demanet and L. Ying, Appl. Comp. Harm. Anal. 23(3):368-387, 2007; [6] A. Manduca et al., Med. Imag. Anal. 7(4):465-473, 2003.