Robust estimation of T_1 and T_2 parameters from complex datasets

Umesh Rudrapatna[1], Annette van der Toorn[1], and Rick Dijkhuizen[1]
[1]*Biomedical MR Imaging and Spectroscopy Group, Image Sciences Institute, University Medical Center Utrecht, Utrecht, Netherlands*

Introduction: Quantitative MRI serves as the bedrock of nearly all MR research. Any possible improvements in the measurement of sample dependent, but scanner independent parameters like T_1, T_2 is beneficial for understanding the underlying biological processes. Though MR data are obtained in complex format, most parametric mapping methods are performed on magnitude information. This may not be an issue at high SNRs, where noise shows near Gaussian characteristics. However, cross comparison of parameters obtained at different SNRs within and across scanners is a common scenario, and thus using unbiased mapping techniques is crucial. Most mapping methods use a least-squares approach to fit data, wherein, asymptotic unbiasedness is assured only under Gaussian noise. It is known that at low SNR, this assumption is violated, and biased estimates of the parameters are obtained.[1] However, since correcting for this bias requires knowledge of noise variance[1], which can only be estimated, these corrections are rarely applied. If parameters could be estimated directly from complex data, asymptotic unbiasedness can be achieved. For this, one would have to account for phase in complex MR data during the fitting procedure. Since phase wrapping (in space) occurs commonly, the perceived signal polarity in space can change sign.[2] Besides, the use of optimization routines that estimate signal magnitude simultaneously with phase is further complicated by the fact that phase perturbations around $\pi/2$ boundaries would require signal polarity changes for smooth convergence. Thus, fitting to complex data is seldom pursued. Instead, complex data are phased and only data in real channel is retained. Though advantageous, this involves costly phase-unwrapping routines which too can fail at low SNR. Besides, field inhomogeneities can perturb the phase maps. These constraints have hindered the estimation of parametric maps by fitting complex data. Below, we discuss a possible way out of this situation by decoupling the estimation of phase and magnitudes in the fitting procedure and thus benefit from unbiased estimates. Although our discussion centers on T_1 and T_2 (and equivalently T_2*) estimation, this procedure may be useful in other circumstances too.

Technique: First, we note that most estimation problems in MR (T_1, T_2, diffusion etc) are separable in nature. i.e., the problem is linear in proton density, but non-linear in other parameters (T_1, T_2, phase etc). This provides us with a choice of using a separable least-squares[3] approach to fit the data. This technique successfully decouples the magnitude and phase estimation problems. Thus, one could use a nested optimization structure, wherein a non-linear optimization estimates the non-linear components by internally estimating the proton density using linear least-squares. This procedure gives a free reign to polarity of proton density, as it is estimated after an update of the non-linear parameters and not along with them. Thus, the ambiguity in phase will be reflected as signal polarity changes. This procedure would not only yield better estimates of the parameters, but in the process, result in robust phase maps too. For estimation of T_2, underlying field inhomogeneities can impart additional phase changes. These can also be modeled in this procedure. Such a T_2 complex data fitting routine can thus yield a field-inhomogeneity map as a by product.

Methods: Simulations were performed to study the benefits of the proposed procedure in T_1, and T_2 estimation. Complex MR data were generated with eqn(1) for T_2 and eqn(2) for T_1 data. For simulating T_2 data, 5 T_2 values, representative of those found in rodent brains at 4-11T were used (10, 20, 40, 60 and 80ms) and TE was set to 12ms. Twelve equally spaced echoes (n=1…12) were assumed. Phase was uniformly randomly chosen between $-\pi$ and π. Five SNR scenarios were simulated with Gaussian noise standard deviation in both real and imaginary channels chosen such that SNR (A/σ) varied from 40 to 5. The field inhomogeneity term, f, was chosen from a Gaussian distribution with a standard deviation of 5 Hz. 10,000 voxel data were simulated for each of the SNR and T_2 value. 3 different fitting algorithms were used to estimate the parameters. For estimations with magnitude data, two techniques were used. The first was a linear fit on the logarithm of the magnitude. The second was a separable least-squares algorithm preformed on the magnitude data. Only estimates of A and T_2 were obtained in these cases. The complex data were fit using a separable least-squares algorithm that was designed to estimate A, T_2, ϕ and f. This routine employed a simplex search for the non-linear terms. T_1 data were simulated in a similar manner, with T_1 values 0.5, 1, 1.5, 2 and 2.5s. T_I was an array of 149 equally spaced values ranging from 56ms to 7.16s, typically used in our Look-Locker experiments. Given that we had more points to fit in time, the SNRs for the simulations used were 10, 5, 3, 2 and 1. Separable Least-Squares algorithms similar to those used in T_2 were built for estimating the parameters from magnitude and complex data. These fitting procedures were also performed on real data obtained from 4.7T (T_2) and 9.4T (T_1) scanners to verify their efficacy. We report results from only non-linear T_2 estimators since they outperformed the linear method.

Results: Fig.1 shows the estimated T_2 values at 3 SNRs for simulated T_2 of 60, 40 and 20ms. Fig.2 shows the estimated T_1 values at 3 SNRs for simulated T_1 values of 1, 1.5 and 2s. We observe that the complex fits to the data yielded nearly unbiased results even at low SNRs and also showed lower variance at each of the SNRs in both cases. Fig 3a-c show results (T_2, phase and field map) obtained by performing fits on complex data obtained from spin echo scans of a rat brain at 4.7T. Fig 4a and b are T_1 maps estimated from a low SNR Look-Locker acquisition of an ex-vivo rat brain at 9.4T. We clearly see the advantage of complex fitting (4a) in comparison to results obtained from a 3 parameter fitting routine (4b) used on magnitude data, as the latter grossly underestimates the T_1 values, as predicted by the simulations.

Discussion and Conclusion: We find that estimations from complex data yielded nearly unbiased results even at low SNRs and also showed lower variance at each of the SNRs compared to estimates obtained from magnitude data. Only partial results from the simulations are shown here, since the trends remained similar at other SNRs and T_1 and T_2 values. The choice of separable least squares to fit even the magnitude data in simulations should be seen in the light that this techniques has been shown to be more robust than conventional non-linear routines which fit all the parameters simultaneously.[3] Since the proposed method assumes that the phase at a given voxel location remains unperturbed during the acquisition of a dataset, unexpected phase deviations can affect these results. Barring such scenarios, many problems in MR can be modeled in a separable sense, and thus, the application of this technique may find wider acceptance.

References: 1)Sijbers et al.,IEEE Tran Med Img,1998;17,357-361 2)Gowland et al.,1991;MRM:18,224-231 3)Steven Kay, Fundamentals of Statistical Signal Processing

$$S(n) = Ae^{-\frac{TE(n)}{T_2}} * e^{i\theta}, \theta = \phi + 2\pi fTE(n) \quad (1)$$

$$S(n) = A\left(1 - (B/A) * e^{(\frac{B}{A}-1)\frac{T_I(n)}{T_1}}\right) * e^{i\theta} \quad (2)$$

Fig. 1

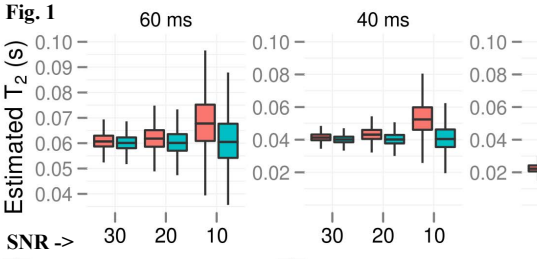

Fig. 2

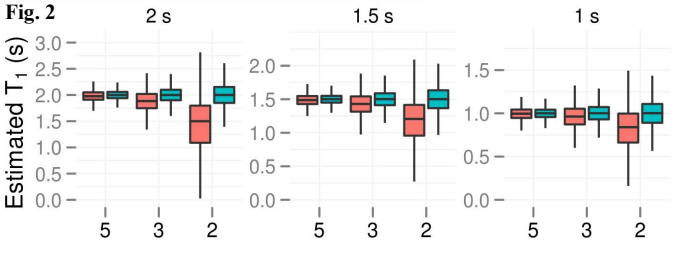

Fig. 1. Results from T_1 simulation studies, Fig. 2. Results from T_2 simulation studies. (x axis, SNR)

3a. Estimated T_2 (complex data fit) from invivo rat brain spin echo scans at 4.7T (scale: 0-100ms) [blue-red] and
3b. Phase (scale: $-\pi$ to π) and 3c. Field inhomogeneity (scale: -4 to 4 Hz) obtained in the same procedure.
4a. Estimated T_1 (using low SNR complex data) from an ex-vivo rat brain at 9.4T (scale: 0-2s) and from a
4b. Routinely used non-linear 3 parameter Look-Locker parameter estimation (from magnitude of same data).