Optimality of Equally-Spaced Phase Increments for Banding Removal in bSSFP

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Introduction The balanced steady-state free precession (bSSFP) pulse sequence is capable of high-SNR imaging in relatively short scan times. However, the off-resonance sensitivity can lead to signal nulls and undesirable banding artifacts in the images. Common methods for eliminating or reducing these banding artifacts involve the acquisition of multiple images with different RF phase increments to shift the bands, and subsequent image combination [1-4] or parameter estimation [5-6] techniques. Virtually all of these techniques utilize phase increments (Δθ) that are distributed evenly between 0 and 2π and here we restrict the investigation to four phases, as commonly used. Figure 1 shows the resulting spectral profiles in the case of four equally spaced phase increments. While equally spaced phase increments seem intuitive because this maximizes the distance between signal nulls, to our knowledge, there has been no previous work to show that this choice is optimal for minimizing signal variations due to off-resonance effects. In this work, we formulate bSSFP banding removal as a parameter estimation problem, and we use the Cramér-Rao Bound (CRB) [7] for the bSSFP signal equation to show that equally spaced phase increments are optimal in the sense that they minimize the worst-case CRB over all possible off-resonances θ.

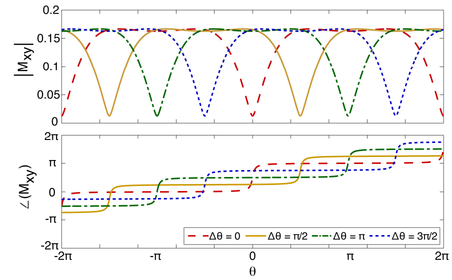

Figure 1. Spectral profiles of four bSSFP sequences with phase cycling Δθ = 0 (dashed red), Δθ = π/2 (solid yellow), Δθ = π (dot-dashed green), Δθ = 3π/2 (dotted blue). Phase cycling yields translation of the magnitude profiles (top) and phase profiles (bottom), with additional phase offsets in each of the phase profiles.

Theory The bSSFP signal equation (Eq. 1) [8] can be fitted to bSSFP images acquired using multiple phase increments to estimate the model parameters. KM_0, which is independent of the off-resonance θ and therefore free of banding artifacts, can be estimated. In addition, an estimate of the off-resonance map θ, as well as estimates of T_1 and T_2, can also be obtained, given that the flip angle α is known. It should be noted that the flip angle α cannot be estimated. The CRB gives the lower bound of the variance of the parameter estimates for any unbiased estimator. It is therefore of interest to minimize, in some sense, the CRB with respect to the experimental setup. Here, we consider finding the optimal vector of phase increments (Δθ) for the acquisition. We resort to a numerical evaluation of the CRB due to the high complexity of the analytical expression. Because the CRB is a function of T_1, T_2, off-resonance (θ), flip angle, and KM_0, which are specific to each scan, we suggest a worst-case scenario optimization. The approach is to minimize the maximum CRB of KM_0 over all θ, T_1, T_2, α, and KM_0. However, empirically it seems that the optimal Δθ only depend on θ, and thus, assuming that θ is unknown, it is sufficient to minimize the CRB over all θ∈ [0, 2π],

$$S_n = KM_0 c e^{i\theta \mathrm{TE}/\mathrm{TR}} \frac{1 - a e^{-i\theta_n}}{1 - b\cos(\theta_n)}$$
$$a = E_2$$
$$b = E_2 \frac{1 - E_1\cos\alpha - E_1 + \cos\alpha}{1 - E_1\cos\alpha - (E_1 - \cos\alpha)E_2^2}$$
$$c = i e^{-\mathrm{TE}/T_2} \frac{(1 - E_1)\sin\alpha}{1 - E_1\cos\alpha - (E_1 - \cos\alpha)E_2^2}$$
$$E_1 = e^{-\mathrm{TR}/T_1}$$
$$E_2 = e^{-\mathrm{TR}/T_2}$$

Eq. 1. bSSFP signal equations.

$$\Delta\theta_{opt} = \arg\min_{\Delta\theta}\left(\max_{\theta} CRB_{KM_0}(\Delta\theta, \theta)\right) \quad \text{(Eq. 2)}$$

Methods Given that the optimal phase increments only depend on θ, we can keep the remaining parameters fixed. Due to the many local minima of the problem in Eq. 2, the CRB function is gridded over all phase increments Δθ and θ, and a 5D exhaustive search is conducted.

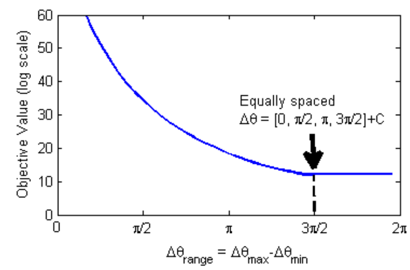

Figure 2. Minimax objective value (Eq. 1) as the allowable range of Δθ (i.e., $\Delta\theta_{max}$- $\Delta\theta_{min}$) is increased from 0 to 2π. The minimum is attained when the allowable Δθ range is 3π/2, and the optimal point is $\Delta\theta_{opt}$ = [0, π/2, π, 3π/2] + C, where C is any constant.

Results We show that when using four phase increments, the optimal values from a minimax point of view are uniformly spaced: $\Delta\theta_{opt}$ = [0, π/2, π, 3π/2] + C, where C is any constant. Figure 2 shows that by restricting the range of Δθ-values to less than 3π/2, the CRB increases. For a range larger than 3π/2 the CRB is constant and the optimum corresponds to equally spaced phases. Figure 3 shows the distribution of the CRB values (normalized by min(CRB)) over all choices of θ and Δθ on the grid. The worst-case CRB using equally spaced phases (i.e., the solution of Eq. 1) is shown in red. Interestingly, the worst-case CRB using equally spaced phases is close to the optimal θ-dependent CRB. Figure 3 is based on a special case, and although the optimal phases will not change with varying KM_0, T1, T2 and alpha, the CRB distribution will. However, in general the minimax performance is similar to what is shown in Fig.3.

Discussion The result might be intuitive, since a periodic signal with an unknown phase shift is measured: it gives a uniformly distributed signal power in terms of probability, and maximizing the distance between the phases minimizes their correlation. However, in general, uniform sampling does not need to be worst-case optimal, and it is therefore important to show that these acquisition phases have theoretical support. It is interesting to note that if prior information regarding θ is available, finding a non-uniform spacing of the phases that gives a lower CRB is possible, but the performance gain is likely to be small. Although the optimization goal was to minimize the maximum CRB of KM_0, the equally spaced phases also minimize the variance of the other model parameters, meaning that the choice is not application-specific.

Conclusion We showed that the (1) worst-case performance in terms of the CRB is minimized when using equally spaced phases, and (2) the resulting performance is relatively close to the optimal performance, had the true value of θ been known *a priori*.

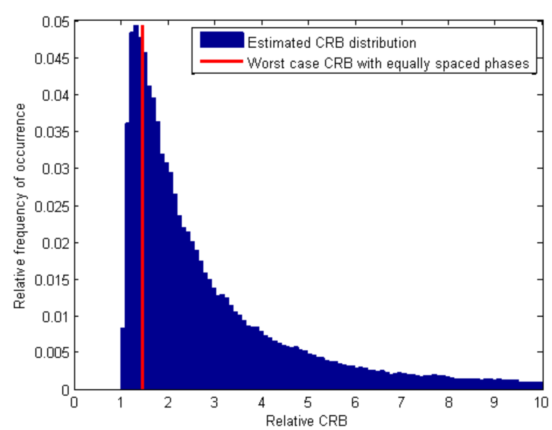

Figure 3. Estimated distribution of CRB values, normalized with min(CRB), for all possible phases Δθ and off-resonances θ, together with the worst-case CRB over all θ for equally spaced phases.

References

[1] Bangerter, *et al.* MRM 51:1038-1047, 2004. [2] Haacke, *et al.* Radiology 175:545-552, 1990. [3] Xiang, *et al.* Proc. 18th ISMRM 74, 2010. [4] Hoff, *et al.* Proc. 19th ISMRM 2824, 2011. [5] Santini, *et al.* Proc. 18th ISMRM 3089, 2010. [6] Björk, *et al.* Proc 19th EUSIPCO 1000-1004, 2011. [7] Stoica *el al.*. Spectral Analysis of Signals, 2005. [8] Freeman, *et al.* JMR 4:366-383, 1971.