

# Saturation Pulse Design with Explicit Sensitivity Maximization for Bloch-Siegert $B_1^+$ Mapping

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**Purpose:**  $B_1^+$  mapping using the Bloch-Siegert (BS) shift [1] is challenging at 7 T and higher, due to SAR limitations and signal loss resulting from the high amplitude and long duration of the saturation pulse (e.g. 4 - 6 ms Fermi, or 4 ms Quad waveforms [2] with  $B_1^{\text{peak}} \approx 20\mu\text{T}$ ). Hence, it is desirable to shorten the saturation pulse while maximizing the sequence's sensitivity (i.e. the derivative of the phase shift as a function of  $|B_1^+|$  ( $\phi_{\text{BS}}$ )) and negligibly exciting on-resonant spins. A previous method for BS pulse design was based on optimization of an indirect and approximate sensitivity objective [1,2]. Here we propose a novel way of designing BS pulses, which directly maximizes sensitivity subject to a constraint requiring negligible on-resonant excitation.

**Methods:** The ideal BS pulse maximizes the sequence's sensitivity and limits the excitation of on-resonant spins to negligible levels. Moreover, maximizing the derivative of  $\phi_{\text{BS}}$  (i.e. sensitivity) with respect to  $|B_1^+|$  and requiring the derivative to be positive across the  $|B_1^+|$  range of interest guarantees a one-to-one mapping of  $\phi_{\text{BS}}$  onto a  $B_1^+ - \Delta B_0$  hyper-plane, as the positivity condition enforces monotonicity of the function  $\phi_{\text{BS}}(|B_1^+|; \Delta B_0)$ . We therefore design the BS waveform to maximize the sum of  $\partial\phi_{\text{BS}}/\partial|B_1^+|$  values evaluated on a  $B_1^+ - \Delta B_0$  grid. This objective function and the on-resonant excitation constraints ( $|\beta| < 0.01$ ) were expressed explicitly in terms of spinors  $\alpha$  and  $\beta$  [3]. The optimization was performed on a  $B_1^+ - \Delta B_0$  grid defined by a range of 0.1-20  $\mu\text{T}$  and  $\pm 600\text{Hz}$  in  $B_1^+$  and  $\Delta B_0$  directions with  $1\mu\text{T}$  and  $60\text{Hz}$  steps, respectively [2]. A 2ms duration was selected and the waveform was treated as a train of 312 block sub-pulses (each  $6.4\mu\text{s}$  long), each being subject to amplitude and phase modulation. To avoid convergence issues due to phase wrapping, the sub-pulses' RF amplitudes were constrained to be between  $\pm 1$  and their phase was unconstrained.

The maximization problem was solved using Matlab's `fmincon` function (The Mathworks, Natick, USA). This highly non-convex optimization problem was initialized with a 2ms Fermi pulse with constant off-resonance term set to  $\pm 4\text{kHz}$ . The optimized 2ms waveform was compared to 2ms and 6ms Fermi pulses with a constant off-resonance shift of  $\pm 4\text{kHz}$ . In order

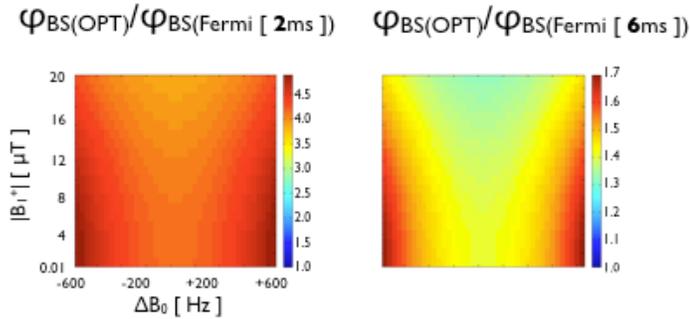


Fig.2: BS phase gain ( $\phi_{\text{BS}}$ ) for 2ms optimized pulse over 2ms and 6ms Fermi pulses. 4-5 fold gain over 2ms Fermi and 40-70% gain over 6ms Fermi pulses clearly show the advantage of the proposed pulse design.

respectively. Figure 2 shows the BS phase gain of the optimized waveform compared to 2ms (left) and 6ms (right) Fermi pulses with a constant 4kHz offset. Finally, the 2ms waveform had a constant block amplitude modulation and the phase modulation function shown in Figure 3 (red) along with the phase modulation of the 2ms Fermi pulse (blue).

**Discussion:** What distinguishes our approach from previous work [2] is the setup of optimization problem, in that we maximize the sequence's sensitivity directly, and use a non-convex optimization algorithm to seek the global maximum of the sensitivity objective. We do not make any approximations regarding the functional dependence of BS phase on  $|B_1^+|$  or  $\omega_{\text{RF}}(t)$ . The optimized waveform achieves the same sensitivity and excites as little on-resonant magnetization as a much longer Fermi pulse. This will improve the robustness of this  $B_1^+$  mapping method and opens doors to new applications. The interesting analytic shape of this waveform (constant amplitude modulation and smooth non-linear phase modulation (arrows in Figure 3)) will be a subject of further investigation. **Acknowledgments:** This work was supported by NIH grant number RO1EB000461. **References:** [1] Sacolick et al., MRM 63: 1315, (2010); [2] M.M. Khalighi, et al., Proc. Intl. Soc. Mag. Reson. Med. 19 4431 (2011). [3] J. Pauly et al., IEEE Trans. on Med. Imag., 10 53 (1991)

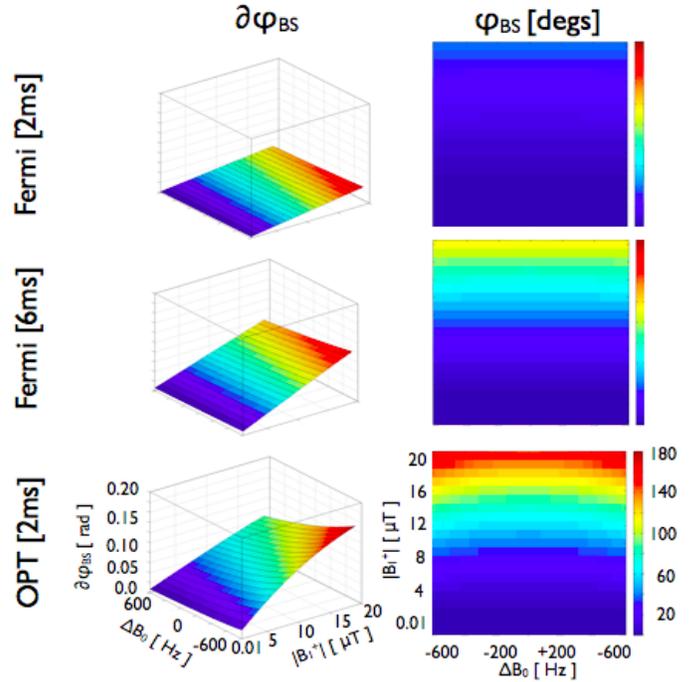


Fig.1: Sensitivity maps (on  $B_1^+ - \Delta B_0$  grid) – left column (expressed in terms of derivative of  $\phi_{\text{BS}}$  with respect to  $|B_1^+|$ ) and phase maps  $\phi_{\text{BS}}$  – right column for 2ms, 6ms Fermi and 2ms optimized pulse.

to make a comparison with previous work possible [2] we show values of BS phase as a function of  $B_1^+ - \Delta B_0$  within the range defined above. Additionally we show values of  $|\partial\alpha|$  and the amount of excitation ( $|\beta|$ ) on the grid of  $B_1^+$  and  $\Delta B_0$  (defined above).

**Results:** Values of  $|\partial\alpha|$  (left column) together with  $\phi_{\text{BS}}$  values (right column) are shown in Fig. 1 for all of the pulses considered here. The maximum  $|\beta|$  values on the  $B_1^+ - \Delta B_0$  grid for the 2ms, 6ms Fermi and 2ms optimized waveform are 0.025/0.028/0.010,

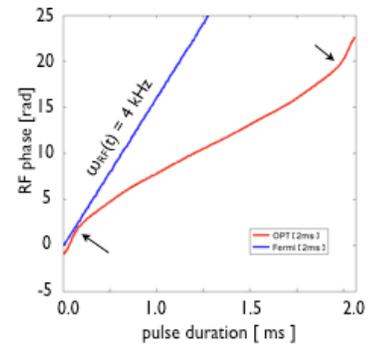


Fig.3: RF phase modulation of 2ms Fermi with a constant 4kHz offset (blue) and 2ms optimized waveform (red). Arrows point to non-linear behavior of the phase at the beginning and the end of the waveform.