

# A QUANTITATIVE STUDY OF SODICKSON'S PARADOX

Yu Ding<sup>1</sup>, Hui Xue<sup>2</sup>, Ti-chiun Chang<sup>3</sup>, Christoph Guetter<sup>3</sup>, and Orlando Simonetti<sup>1</sup>

<sup>1</sup>Dorothy M. Davis Heart and Lung Research Institute, The Ohio State University, Columbus, OH, United States, <sup>2</sup>Siemens corporate research, <sup>3</sup>Siemens Corporate Research

**Introduction:** GRAPPA, a k-space based pMRI technique, is widely used clinically for dynamic cardiac MRI because of its robustness [1]. It reconstructs the missing k-space by applying a convolution kernel which is estimated from auto-calibration signal (ACS) lines using linear regression. Intuitively, ACS lines with higher SNR should boost the accuracy of the kernel estimation, and increase the SNR of GRAPPA reconstruction. Paradoxically, Sodickson and his colleagues pointed out that a higher SNR in the ACS lines used in GRAPPA may lead to lower SNR in the reconstructed images [2,3]. This abstract presents a quantitative study of how the noise in the ACS lines affects the SNR of the GRAPPA reconstruction, and based on this proposes a simple solution to improve the SNR of TGRAPPA. We hypothesize that the SNR of the ACS lines is proportional to the condition number of the linear regression equation in the GRAPPA kernel estimation. Therefore, the GRAPPA kernel estimated from higher SNR ACS lines amplifies the random noise in GRAPPA reconstruction. In TGRAPPA reconstruction of dynamic image series, a widely used method to acquire ACS lines is to average-all-frames (AAF). This approach may be sub-optimal because it results in ACS lines with very high SNR. Instead of averaging, we propose to tile all frames (TAF) as ACS lines to improve the SNR of the reconstructed images.

**Methods:** The hypothesis was tested in a phantom study. We acquired SSFP real-time cine images on a 1.5T MR scanner (MAGNETOM Avanto, Siemens Healthcare, Germany) using an SSFP sequence. 14 images series each with 86 frames were acquired with flip angles varying from 1° to 70°. All k-space was fully sampled. 18 sets of ACS lines with variant SNR were prepared. The first 14 sets of ACS lines were the first frame of the 14 image series, and the SNR was varied by changing flip angle. The last four sets of ACS lines were prepared by averaging 2, 4, 8, and 16 frames of the 70° flip angle cine, thus the SNR was varied by changing the noise level. A 32-channel cardiac array coil (RapidMRI, Columbus, OH) was used for data acquisition. The root-mean-square-error (RMSE) of GRAPPA reconstruction using kernels estimated from ACS lines with variant SNR was calculated.

We also acquired SSFP real-time cine images in vertical and horizontal long-axis and one short-axis views in one healthy volunteer on a 3.0T MR scanner (MAGNETOM Trio, Siemens Healthcare, Germany) using TGRAPPA with parallel acceleration rate = 6. Imaging parameters were: 192 × 96 reconstructed matrix, 6 mm thick slice, flip angle=48°, TE/TR = 1.0/2.56 ms, pixel bandwidth=1447 Hz/pixel, FOV = 380 × 285 mm<sup>2</sup>. A total of 256 images were acquired per image series to provide sufficient data for statistical analysis. TGRAPPA kernels were estimated using AAF and TAF methods, and two image series were reconstructed respectively. Please refer to Figure 1 for two estimates of ACS lines from the same raw data. The image SNR was measured using the MP-law method [4]. We use a 4 × 5 GRAPPA kernel in all reconstructions.

**Results:** Figure 2a shows that the condition number follows a linear relation with SNR over two orders of magnitude. Figure 2b demonstrates that the RMSE of GRAPPA reconstruction reaches the minimum when the ACS lines have moderate SNR. The “sweet spot” corresponds to ACS lines with flip angle = 40°. Measurements performed on the volunteer image data indicate that the TAF method improved the TGRAPPA reconstruction SNR by more than 10% in all three cardiac views.

**Discussion and Conclusion:** In conclusion, this study reveals a simple proportional relation between the SNR of ACS lines and the condition number of the GRAPPA kernel regression equation. Counter intuitively, high SNR ACS lines should not be used to avoid the SNR degradation of the GRAPPA reconstruction. Higher SNR in the ACS lines has similar effects as applying a lower Tikhonov regularization parameter in linear regression, and vice versa. Then higher SNR of the ACS lines may amplify random noise in GRAPPA reconstruction, but preserve the image sharpness; lower SNR of the ACS lines may boost image SNR but over-smooth the image. In real-time cardiac cine images acquired using the TGRAPPA technique, the SNR of ACS lines estimated using AAF may be too high. The estimation of ACS lines using TAF is a better strategy in TGRAPPA reconstruction because it uses all k-space data as ACS lines without affecting the SNR. The in vivo experiment shows that this new approach improves TGRAPPA SNR in a clinically relevant imaging protocol.

**References:** [1] Griswold MA, et al, Magn Reson Med, 47:1202 (2002). [2] Sodickson DK, et al. Magn Reson Med, 44: 243 (2000). [3] Ernest NY. et al, Magn Reson Med, 53: 1383 (2005). [4] Ding Y et al, Magn Reson Med 63: 782 (2010).

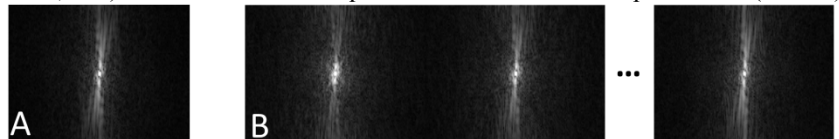


Figure 1. ACS lines of one channel. A: The ACS lines estimated by averaging all acquired frames resulting in high SNR. B: ACS lines estimated by tiling all acquired frames together. Each frequency encoding line spans multiple full k-space.

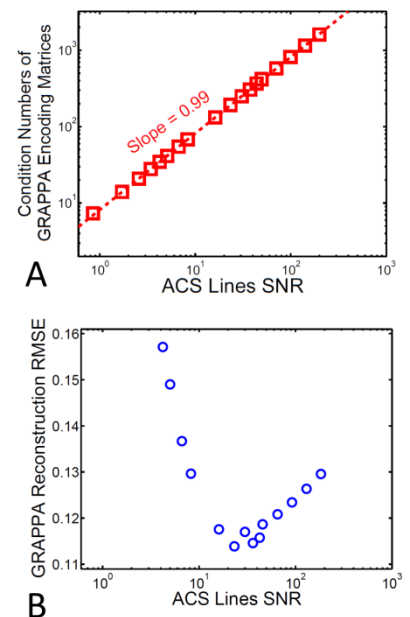


Figure 2. A: The condition number increases linearly with ACS lines SNR. B: GRAPPA. the RMSE of the GRAPPA reconstructed has a minimum at moderate ACS lines SNR, corresponding to the flip angle = 40°.