A Nonlinear ARMA Model for GRAPPA Reconstruction

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INTRODUCTION:
GRAPPA [1] reconstructs the missing *k*-space data by a linear combination of the acquired data using a set of coefficients obtained through calibrations. The conventional GRAPPA can be considered as a finite impulse response (FIR) or equivalently a moving average (MA) model. Recent work [2,3] attempts to improve GRAPPA using an infinite impulse response (IIR) or equivalently an auto-regressive moving average (ARMA) model. However, the linear ARMA model has some limitations such as the outliers lead to estimation errors [4] and poor AR estimation severely affects the estimation accuracy of the MA part [5]. In this abstract, a novel method using nonlinear ARMA (NLARMA) model is proposed to address the issue in IIR GRAPPA reconstruction. The method is motivated by the success of nonlinear FIR model for GRAPPA [6]. The proposed NLAMRA model improves over the linear MA model used in conventional GRAPPA by incorporating both recursion and nonlinearity. The experimental results demonstrate that the proposed method is able to improve the reconstruction quality of the conventional GRAPPA and IIR GRAPPA in suppressing noise and artifacts.

THEORY AND METHOD:
The conventional GRAPPA is a linear MA process with $\mathbf{y}=\mathbf{W}^{\mathrm{MA}}\mathbf{x}$ (1) and the IIR GRAPPA [3] is a linear ARMA process formulated as $\mathbf{y}=\mathbf{W}^{\mathrm{AR}}\hat{\mathbf{y}}+\mathbf{W}^{\mathrm{MA}}\mathbf{x}$ (2), where $\mathbf{w}^{\mathrm{AR}}$ and $\mathbf{w}^{\mathrm{MA}}$ represent the coefficients for the AR and MA parts respectively, $\mathbf{y}$ denotes the missing *k*-space data, $\hat{\mathbf{y}}$ denotes the estimated missing data obtained from the conventional GRAPPA, and $\mathbf{x}$ represents the vector for the acquired *k*-space data. The noise in *k*-space data causes nonlinear effects in the auto-calibration and reconstruction procedures of GRAPPA and nonlinear models have shown to improve GRAPPA reconstruction [6]. In this study, we propose a nonlinear model to improve IIR GRAPPA and name the method nonlinear ARMA (NL ARMA). Specifically, the linear MA and AR terms in the IIR model are extended to nonlinear functions. Here a second-order polynomial function is used. The calibration/reconstruction equation is given in Eq. (3) on the right,

$$
\begin{aligned}
\hat{s}_j\left(k_y+r\Delta k_y,k_x\right)=&\sum_{l,b,h} w_{j,r}^{\mathrm{MA}}(l,b,h)\times s_l\left(k_y+bR\Delta k_y,k_x+h\Delta k_x\right)+\\
&\sum_{l,(q\neq r,R),h} w_{j,r}^{\mathrm{AR}}(l,q,h)\times \hat{s}_l\left(k_y+q\Delta k_y,k_x+h\Delta k_x\right)+\\
&\sum_{l,b,m,m'} w_{j,r}^{\mathrm{MA}}(l,b,m,m')\times s_l\left(k_y+bR\Delta k_y,k_x+m\Delta k_x\right)\times s_l\left(k_y+bR\Delta k_y,k_x+m'\Delta k_x\right)+\\
&\sum_{l,(q\neq r,R),m,m'} w_{j,r}^{\mathrm{AR}}(l,q,m,m')\times \hat{s}_l\left(k_y+q\Delta k_y,k_x+m\Delta k_x\right)\times \hat{s}_l\left(k_y+q\Delta k_y,k_x+m'\Delta k_x\right)+\\
&\sum_{l,b,m,(q\neq r,R),m'} w_{j,r}^{\mathrm{CROSS}}(l,b,m,q,m')\times s_l\left(k_y+bR\Delta k_y,k_x+m\Delta k_x\right)\times \hat{s}_l\left(k_y+q\Delta k_y,k_x+m'\Delta k_x\right)
\end{aligned}
\qquad (3)
$$

where s_j are acquired *k*-space data, $\hat{s}_l$ are missing data and $w_{j,r}$ are the coefficients to be estimated in calibration and then used in reconstruction.

The proposed approach takes the following two steps iteratively based on Eq. (3). First, all missing data $\hat{s}_l$ are obtained through the conventional GRAPPA reconstruction. With both ACS and the estimated missing data, in the second step, the coefficients $\mathbf{w}^{\mathrm{AR}}$, $\mathbf{w}^{\mathrm{MA}}$, and $\mathbf{w}^{\mathrm{CROSS}}$ can be obtained by solving a set of linear equations. Then, the estimated coefficients are plugged back into Eq. (3) to update the values for the missing data. In our experiments, we found that linear terms of AR part deteriorate reconstruction quality, so this term is not used in our final model. Although a nonlinear ARMA model is used here, Eq. (3) is still a linear function of the unknown coefficients. Least squares method was used to solve the equation without any regularization.

RESULTS AND DISCUSSION
We tested the proposed method on both phantom and *in vivo* data sets. An 8-channel phantom dataset was acquired using a Gradient Echo sequence (TE/TR = 10/100 ms, 31.25 kHz bandwidth, matrix size = 256×256, FOV = 250 mm^2) and an 8-channel brain dataset was obtained using a 2D spin echo (SE) sequence (TE/TR = 11/700 ms, matrix size = 256 × 256, FOV = 220 mm^2). The data were acquired with Nyquist rate and used for reconstructing the reference image. The data were then retrospectively undersampled with an outer reduction factor (ORF) of 6 and 36 ACS (net R=3.51) for phantom and ORF of 5 and 48 ACS (net R =2.81) for brain. Reconstructions using GRAPPA, IIR GRAPPA and the proposed NL ARMA method are shown in Figure 1. In consistence with the observation in [2], IIR GRAPPA does not show significant improvement over the conventional GRAPPA at high outer reduction factors. Both methods present large noise and artifacts. In contrast, the proposed method improves the reconstruction quality by suppressing noise and aliasing artifacts. The computation time of the proposed method is similar to that of IIR GRAPPA, which is about 2-4 times of the conventional GRAPPA.

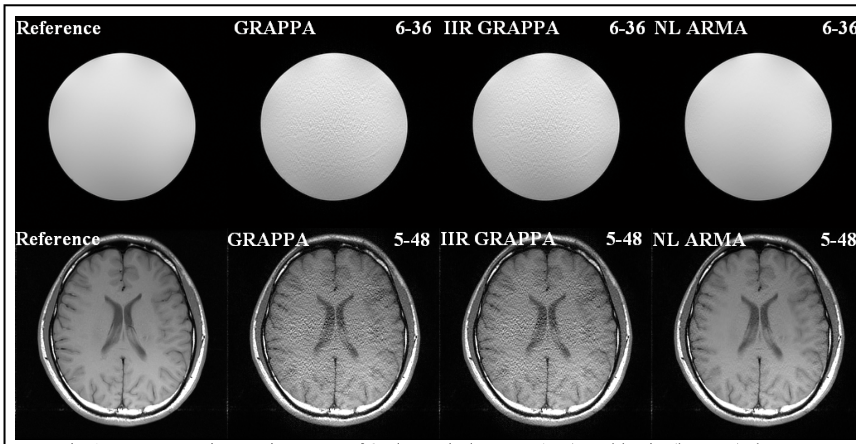

Fig.1 Reconstructions using a set of 8-channel phantom (top) and brain (bottom) datasets.

CONCLUSION: We have presented an improved GRAPPA method using a nonlinear ARMA model. Experimental results demonstrate that the method can effectively reduce the noise and aliasing artifacts in conventional GRAPPA and IIR GRAPPA.

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