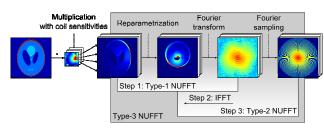
## Reconstruction of undersampled radial PatLoc imaging using Total Generalized Variation

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Introduction: It was recently demonstrated that radial sampling can be generalized to imaging with nonlinear non-bijective encoding fields (NB-SEMs) [1], commonly known as PatLoc imaging [2]. Benefits of radial sampling like low sensitivity to object motion and inherent autocalibration for parallel imaging due to oversampling of k-space can also be observed in the case of nonlinear encoding fields. A prominent star-shaped artifact has been observed in [1], if a spin distribution is encoded with pure quadrupolar SEMs and a sub-sampled radial trajectory. This artifact is particularly problematic, if the images are intermediately reconstructed onto the Fourier-domain of the PatLoc k-space (termed PatLoc-encoding space in [3]). It could also be shown in [1] that image reconstruction directly onto the final image space reduces the artifact, however, without eliminating it. The goal of the work presented here was to eliminate the prominent star-shaped artifact up to high acceleration factors in the context of PatLoc image encoding. We propose to use iterative image reconstruction with variational constraints, which has already proven effective for reduction of streaking artifacts in sub-sampled radial trajectories with linear SEMs [4,5].



**Fig. 1:** Flowchart of the forward sampling operator *F*. A description of the individual steps is given in the text.

**Theory:** Iterative image reconstruction with a constraint that is based on the second order total generalized variation  $(TGV^2)$  [5] was used in this work. With this approach, the following optimization problem has to be solved:

$$\min ||F(u) - k||_2^2 + \lambda \cdot TGV^2(u).$$

In this equation, F is the forward sampling operator, u is the reconstructed image, k is the measured k-space data,  $\lambda > 0$  is a regularization parameter and  $TGV^2$  is the penalty functional which is described in detail in [5]. To describe F, first note that for encoding with linear gradients, the forward sampling operator consists of the multiplication with the coil sensitivity profile and a 2D non-uniform FFT (Type-2 NUFFT) [6] mapping data on a Cartesian grid to the Fourier transform evaluated at the radial sampling trajectory. On the other hand, for regularly sampled PatLoc imaging, i.e., encoding with non-linear gradients and evaluation of the data on a Cartesian grid [2,3], the PatLoc sampling operator is not a Fourier transform. However, its adjoint corresponds to a Type-2 NUFFT which takes k-space data on a Cartesian grid, computes its inverse Fourier transform and reparametrizes to the non-Cartesian grid associated with the NB-SEMs. The PatLoc sampling operator is the adjoint of this operator (also known as Type-1 NUFFT), in which the reparametrization is inherent. In radially sampled PatLoc imaging, these two operations are combined (Type-3 NUFFT). The forward sampling operator F is therefore composed of the multiplication with the coil sensitivity profile followed by the Type-3 NUFFT. The latter is realized by performing aforementioned Type-1 NUFFT, an inverse FFT and the Type-2 NUFFT associated with the radial sampling trajectory. This is illustrated in Fig. 1.

Methods and Results: A simulation based approach, using the Shepp-Logan phantom (matrix 256×256), was used to test the proposed reconstruction method. An additional smooth modulation was applied to the image because the Shepp-Logan phantom only consists of a small number of piecewise constant regions, which is a best case scenario for bounded variation methods and might lead to overly-optimistic results. It was multiplied with measured coil sensitivities from an 8 channel head coil and the forward PatLoc operator was used to generate data in PatLoc k-Space with 64, 32 and 16 radial projections. The results of reconstructions with Tikhonov CG adapted from [7], using min-max interpolation NUFFT from [6], and the proposed TGV² based approach are shown in Fig. 2. Further experiments were performed with in-vivo measurements on a 3T Tim Trio (Siemens, Germany) with a modified encoding hardware for brain imaging using PatLoc coils that generate two orthogonal quadrupolar SEMs [8]. Ethics approval and written consent were obtained prior to the experiment and measurements were carried out according to safety considerations for in-vivo PatLoc imaging [9]. A conventional 8 channel receive coil was used, and sensitivity

profiles were post-processed based on [10]. Spin-echo images with TR/TE=500/11ms were acquired with a conventional radial sequence, where the generated currents were applied to the PatLoc coils instead of the standard gradient coils. Projections with 256 samples along each readout were acquired with alternating currents between successive projections to distribute the effects of trajectory inaccuracies more evenly. A sub-sampled dataset was mimicked by taking a reduced number of projections from the full dataset. Reconstructions with Tikhonov CG and  $TGV^2$  from 103, 52, 35 and 26 radial projections are displayed in Fig. 3.

Discussion: The results demonstrate that radial imaging with nonlinear encoding fields and TGV<sup>2</sup> regularization yield reconstructions without residual aliasing artifacts, even at very high reduction factors. In the simulations, artifact-free reconstructions were obtained with as few as 16 projections, corresponding to an effective acceleration of more than 25 compared to a fully sampled radial trajectory. It must be noted that even with the additional smooth signal changes, the Shepp-Logan phantom is still well suited for bounded variation methods, which is the reason why it was possible to obtain higher acceleration factors than in the in-vivo case of Fig. 3. However, no staircasing artifacts are visible in the reconstructions, in line with the results of [5]. Additionally, reconstructions from radial PatLoc data show artificial circular Gibbs ringing structures at the center of the image [1]. The circular structures are caused by the side-lobes of the central PSF that dominates the PSF from neighboring pixels. The dominance is caused by the weak encoding fields at the center with the consequence of prominent signal accumulation at the center and far-reaching signal contamination which compromises image quality in the nearby region. Although not eliminated, the radius of the artifact is reduced significantly in the TGV<sup>2</sup> reconstruction. This shows that in the context of PatLoc, TGV<sup>2</sup> is not only useful to eliminate artifacts resulting from sub-sampled radial trajectories, but also to enhance other properties of the PSF, thereby improving the overall quality of the reconstructed images.

References: [1] Schultz et al., IEEE TMI (DOI: 10.1109/TMI.2011.2164262), [2] Hennig et al., MAGMA 21: 5-17 (2008), [3] Schultz et al., MRM 64:1390-03; [4] Block et al., MRM 57: 1086-98 (2007), [5] Knoll et al., MRM 65: 480-91 (2011), [6] Fessler et al., IEEE TSP 51: 560-74 (2003), [7] Prüssmann et al. MRM 46:638-51 (2001), [8] Welz et al., ISMRM 762 (2009), [9] Cocosco et al., ISMRM 3946 (2010), [10] Walsh et al., MRM 43: 682-90 (2000).

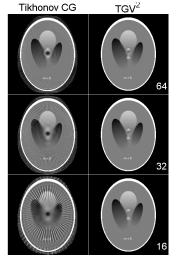
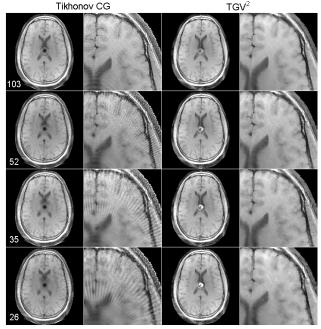


Fig. 2: Radial PatLoc data (modified Shepp-Logan phantom, 256×256) with 64, 32 and 16 projections. Conventional CG with Tikhonov regularization (left), proposed TGV<sup>2</sup> approach (right).



**Fig. 3:** In-vivo radial PatLoc brain measurements with 103, 52, 35 and 26 projections. CG with Tikhonov regularization (left), and the proposed  $TGV^2$  approach (right).