Quantification Error in MREPT due to B1 Map Inaccuracy

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Introduction: Magnetic Resonance Electrical Property Tomography (MREPT) [1,2] was introduced to noninvasively image the distribution of electric conductivity and permittivity in the human body at the Larmor frequency. Of special interest would be ability to measure electric property of blood [3]. Over homogeneous regions, the relationship between the electric properties and magnetic fields can be simplified as a Helmholtz equation of B_1^+. By determining the wavenumber of the Helmholtz equation over the homogeneous region of a tissue, both electric conductivity and permittivity of the tissue can be determined. For a small tissue whose size is much smaller than the wavelength of the B_1^+, the estimate of the conductivity would be very sensitive to noise. However, there have been no previous reports on this matter. In this work we propose methods using B_1^+ maps generated by EM simulations on numerical phantoms to investigate and evaluate how the error in the conductivity estimates using the Helmholtz equation of B_1^+ is related to the SNR of B_1^+ and the size of the tissue. The proposed method was applied for several types of tissues. The EM simulations were performed using Bessel Boundary Matching Method [4] which generates the simulated electric and magnetic fields for cylindrical models in a very short computation time. Our proposed method can be useful in deciding what levels of SNR of B_1^+ are needed to have an acceptable estimate of the conductivity before the B_1^+ map are measured in the scanner.

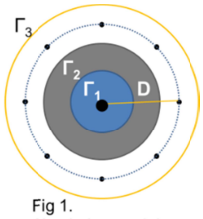

Fig 1. Simulation model (infinite cylinder)

Methods: Simulation Models and Methods: To generate noiseless B_1^+ maps for the tissue of interest satisfying the Helmholtz equation, the numerical phantoms and the coil model shown in Fig. 1 were used to simulate. The numerical phantom with two tissues was designed to be infinitely long cylinders. The tissue of interest was inserted in the middle and the radius of the tissue varied from 2mm to 45mm. To remove the sharp transition of the wavelength at the tissue boundary, a large volume of muscle was added in the model to surround the tissue of interest with the outer radius of 0.2m. The conductivity and permittivity values of the tissues were obtained from [5]. The location of RF coil and RF Shields were fixed at the radius of 0.3m and 0.35m driven by eight rod quadrature coil at four frequencies, 21MHz (0.5T), 64MHz (1.5T), 128MHz (3T), 300MHz (7T). The axial slice of noiseless B_1^+ maps was acquired with a spatial resolution of 1mm*1mm.

$$\hat{\kappa}(r) = \frac{\oint_{\Gamma_1} \nabla B_1^+ \cdot d\boldsymbol{a}}{\omega^2 \mu \int_V B_1^+ dV} \quad \cdots(1), \quad \kappa(r) = \varepsilon(r) - i\sigma(r)/\omega \quad \cdots(2)$$

$$\hat{e} = \frac{\sqrt{\sum_{i=1}^{N} (\hat{\sigma} - \sigma)^2 / N}}{\sigma} \times 100 \quad (\hat{\sigma} \text{ is estimated conductivity}) \quad (3)$$

Evaluation of error in conductivity estimates: The error in the conductivity estimates were evaluated for three types of Gaussian noises added to the noiseless B_1^+ map, 1. complex noise in B_1^+, 2. noise in the magnitude of B_1^+, 3. noise in the phase of B_1^+. For the first two cases, the standard deviations of the noises were adjusted to make the SNR of B_1^+ from 10 to 10000. From the noisy B_1^+ map, conductivity of the tissue of interest was determined by Eq. 1 [2], integrating over the homogeneous region of the tissue. At each noise level, the normalized root mean square error (NRMSE) in the conductivity estimates, Eq. 3, was evaluated using Monte Carlo simulations with 2000 trials.

Results: Fig. 2 shows the error in the conductivity estimates of the Blood tissue at four frequencies (21, 64, 128, 300MHz) with the complex Gaussian noises added to the noiseless B_1^+ maps. To be within 10% error shown as dotted lines in Fig.2 with the SNR of 1000 in the B_1^+ map, the minimum radius of the tissue was about 15mm (21MHz), 9mm (64MHz), 6mm (128MHz), 4mm (300MHz). With the same SNR of B_1^+ maps, the high field system allows higher spatial resolution in MREPT. At 128MHz, the SNR of B_1^+ maps required to result within 10% NRMSE in the conductivity estimates were about 8220 for 2mm, 4030 for 3mm, 1350 for 5mm, 670 for 7mm, 360 for 10mm, 180 for 15mm. This results show that the error in the conductivity estimates were inversely proportional to the area of the homogeneous region. In Fig. 3, the error in the conductivity estimates of the Blood tissue at 128MHz are shown for two types of noise in the B_1^+ maps, magnitude error and the phase error to evaluate which error is more influential. Even with SNR of 10 in the magnitude of B_1^+ assuming no error in the phase of B_1^+, the error in the conductivity estimates was less than 5% for 2mm radius. However as shown in Fig. 3(b), the error in the conductivity estimates were more than 20% for the radius of 15mm with the small noise in the phase about 0.01 (0.5°). The conductivity estimates are more sensitive to the noise in the phase of B_1^+. Fig. 4 shows the error in the conductivity estimates for six types of tissues with the radius of 10mm. As conductivity increases, lower SNR in the B_1^+ maps can be used to have the same error in the conductivity estimates. Using Eq. 1, we determined that at 128MHz, the minimum diameter of the homogeneous region required for less than 10% of NRMSE in the conductivity estimates given a single slice of B_1^+ map with the spatial resolution of 1mm and the SNR of 100, were about 300mm (Fat), 100mm (WM), 70mm (GM), 28mm (Prostate), 50mm (Blood), 30mm (CSF).

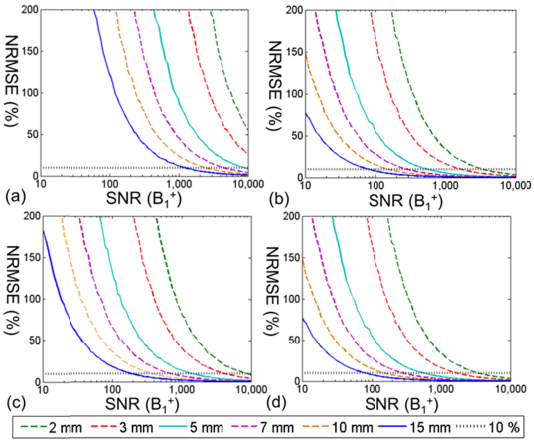

Fig 2. NRMSE of estimated conductivity according to SNR of B_1^+ for different size tissues, (a) 21MHz (b) 64MHz (c) 128MHz (d) 300MHz

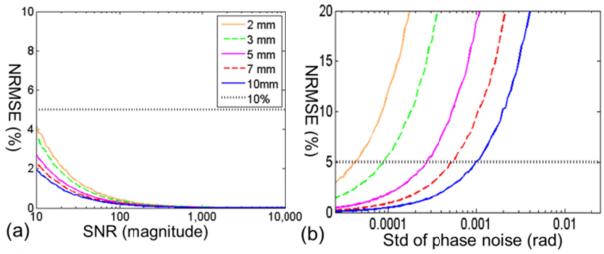

Fig 3. Comparison between (a) Magnitude noise and (b) Phase noise (note : the Y-axis of (a) has different scale for visualizing NRMSE)

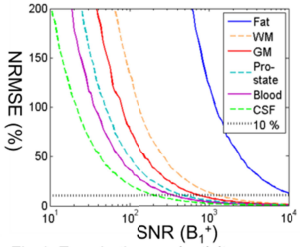

Fig 4. Error in the conductivity estimates for six types of tissues at the radius of 10mm (128MHz)

Discussions & Conclusion: The proposed methods using simulated B_1^+ maps can be used to evaluate the error in the conductivity estimates given the SNR in the B_1^+ and the size of the tissue. In this work, only Eq. 1, which averages over the homogeneous region of a tissue, was used to estimate the conductivity of the tissue. Spatial filtering or other reconstruction methods that might reduce the error were not considered here. With the same SNR in the B_1^+ maps, the NRMSE in the conductivity estimates was decreased as the wavenumber or the size of the tissue increases. In the case of $\sigma/\omega\varepsilon \gg 1$, the conductivity can be determined by the phase of the B_1^+ [2]. For Blood tissue, where $\sigma/\omega\varepsilon \approx 15 \gg 1$, the error in the conductivity estimates was insensitive to the magnitude error in B_1^+, but very sensitive to noise in the phase of B_1^+. **References:** [1] Katscher et al, IEEE TMI, 28:136-75, 2009, [2] Voigt et al, MRM 66:456–466, 2011, [3] Visser, Med. & Biol. Eng & Comput., 30:636-640, 1992, [4] van den Bergen et al, Phys. Med. Biol. 54:1253–1264, 2009, [5] http://niremf.ifac.cnr.it/tissprop/. **Acknowledgements:** Korea MKE and KIAT through the Workforce Development Program in Strategic Technology, KOSEF grant No. 2011-0002495.