# Temporal Tomographic Imaging By Greedy Feteke and Golden Means Sampling

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## Introduction

Electron Paramagnetic Resonance Imaging (EPRI) can provide insight into in vivo anatomic and functional imaging of free radicals and paramagnetic molecules and their role in disease in small animal models. There are many sampling strategies available for EPRI to determine which gradient angles are acquired: equal linear angle (ELA), equal solid angle (ESA), Feteke points, and Monte-Carlo-based sequences [1-3]. ELA is the most common sampling strategy because images can be reconstructed from it very quickly, however, it is well-known to be highly inefficient because approximately 1/3$^{rd}$ of the projections are nearly redundant and concentrated at one of the axes. To date, no sampling strategy has addressed the temporal aspect of EPRI. All of the aforementioned EPRI sampling strategies assume a static object; repeating the same angles over time provides highly redundant and correlated information. Our goal was to develop a more robust sampling strategy and improve the application of EPRI to dynamic imaging. The limited signal to noise ratio in EPRI makes it important to distribute projections uniformly in both space and time. In this work, we define a new temporal sampling strategy called Greedy Feteke (GF), which provides a flexible trade-off between temporal and spatial resolution. We compare GF against a temporal sampling strategy from the MRI projection literature, Golden Means (GM) sampling, which is a generalization of the Fibonacci sequence.

## Methods

We generated N=2,000 Feteke points by minimizing the electrostatic potential (U) defined by a set of point charges on a sphere which repel each other according to Coulomb's law: $U = \frac{1}{2} \Sigma \mid v_i - v_j \mid^{-t}$ for all $1 \le i < j \le N$ (t=1) [4]. This set of points has excellent spatial uniformity but lacks temporal information. To make this sampling strategy useful for dynamic EPRI, we calculated which ordering of N! possible orderings with a greedy algorithm. Starting with an arbitrary point from the precomputed set of all Feteke points, the location of the $i^{th}$ point added is the one which minimizes the increase in U from the set containing $i$-1 points. The point added must be in the precomputed set, and it must not have been sampled yet. We called this "Greedy Feteke," indicating that the order chosen is not guaranteed to be globally optimal among all possible orders, but any subset of the first $i$ points will be as uniform as possible. We hypothesized that this sampling strategy would give the best possible results when only small numbers of projections can be acquired.

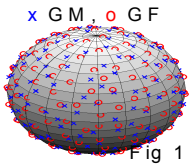

Fig 1

In contrast, Golden Means (GM) sampling is based on a generalization of the Fibonacci sequence, and it has previously been used in MRI projection acquisitions [5]. The sampling is entirely deterministic and given by the formulas $\beta = \cos^{-1}(\mathrm{frac}(m\Phi_1))$ and $\alpha = 2\pi\ \mathrm{frac}(m\Phi_2)$, where m is a positive integer, frac() computes the fractional part of a real number, $\beta$ is the elevation angle from the xy plane towards the +z axis in radians, and $\alpha$ is the azimuthal angle from the +x axis towards the +y axis in radians. $\Phi_1$ and $\Phi_2$ are the solutions to the generalized 2D golden means problem ($\Phi_1 \approx 0.4656$, $\Phi_2 \approx 0.6823$). This distribution never achieves uniformity, but it approximates uniformity fairly well for subsets of any size out of a large group of projections (Fig 1).

We created a dynamic phantom for our 1.2 GHz CW EPRI system with a 6 ml chamber that had two input lines, pumping in 75 mM 4-oxo-2,2,6,6-tetramethylpiperidine-l-oxyl (TEMPONE) and deionized water, respectively. The TEMPONE was diluted by passive mixing inside the chamber, which slowly drained into a waste container. A total of 4000 projections for the GM sampling strategy were obtained while the concentration of TEMPONE was reduced from 10 mM to 0 mM by constant input of water at 5 ml/min ("washout"). The chamber was refilled with 10 mM TEMPONE, and the washout was repeated while projections were acquired using GF sampling. The 3D EPRI acquisition parameters were 0.1 s per projection, 2 mT sweep field width, 0.04 mT center field, 0.07 mT modulation field amplitude, 100 kHz modulation frequency. We also created static images before the temporal acquisitions for comparing SNR between GF and GM. All images were reconstructed using filtered backprojection after typical EPRI processing (hyperfine correction and deconvolution).

## Results

In static conditions, Greedy Feteke has higher SNR than Golden Means for 125 or fewer projections (Fig 2), which is a realistic range for temporal CW EPRI acquisitions. Acquiring projections faster than 0.1 s each quickly degraded SNR to unusable levels. Greedy Feteke followed the timecourse of the washout more accurately than GM (Fig 3(a) vs 3(b)), where the mean signal intensity in a region of interest (ROI) in the center of the reconstructed images was plotted against the first integral of each projection (i.e., the sum of all signal in the resonator). GM had a large temporal lag between 55 s and 250 s; GF appeared to have minimal lag at most times. Images reconstructed from temporal groups of 64 projections had higher SNR and fewer artifacts at most timeframes when using GF sampling as opposed to GM sampling (Fig 4, left vs. right images). In static images, GM and GF both outperformed ELA and ESA sampling for subsets of 200 projections or fewer (results not shown).

## Discussion

Greedy Feteke is better than Golden Means, at least for the dynamic washout phantom reported here. GM maintains an approximate uniformity for any set of projections grouped together in time, but uniformity of larger sets is suboptimal. In contrast, GF for the first $i$ projections always tries to be as uniform as possible for any $i$, which appears to be an advantage for this type of experiment. In vivo applications of GF sampling are in progress, including EPR measurements of myocardial oxygenation, redox state, pH, and nitric oxide metabolism. GF can be applied to other tomographic modalities, such as cardiac MRI with k-space radial sampling. Improved reconstructions may result from applying compressed sensing to this type of EPRI acquisition due to temporal correlations. Future work will include evaluating these sampling strategies during cardiac EPR imaging. The extension of GF to 4D spectral-spatial EPRI is straightforward and compelling, and we anticipate a significant improvement in temporal cardiac EPRI.

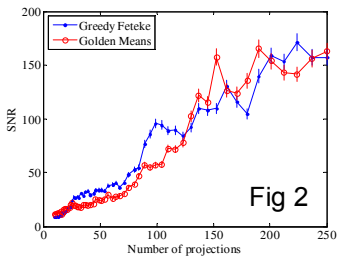

Fig 2

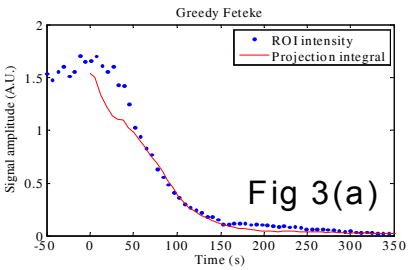

Fig 3(a)

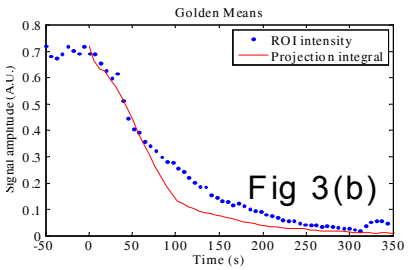

Fig 3(b)

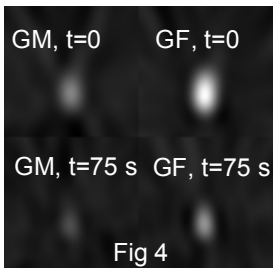

Fig 4

**References and Acknowledgements.** [1] Deng et al. J Magn Reson 2005; 174(2):177-187. [2] Ahmad et al. J Magn Reson 2007; 184(2):236-245. [3] Ahmad et al. J Magn Reson 2007; 187(2):277-287. [4] Ahmad et al. IEEE Conf (ICASSP) 14-19 March 2010 (Dallas, TX): 461-4. [5] Chan et al. MRM 2009; 61(2):354-363. This work was supported by NIH R01 EB004900, and the first author was supported by NIH F32 EB012932.