Multi-Resolution Reconstruction of Mechanical Properties using Non-Linear Inversion MR Elastography.

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$$\nabla\cdot\mu(\nabla\vec{u}+\nabla\vec{u}^T)+\nabla(\lambda\nabla\cdot\vec{u}) = -\rho\omega^2\vec{u}$$

Eq 1: Non-homogeneous form of Navier's Equation, governing motion in a solid with spatially varying viscoelastic properties, where $\vec{u}$ = 3D displacement vector, μ = complex valued shear modulus, λ = compressional modulus, and ρ = density.

INTRODUCTION MR Elastography generates quantitative images of mechanical properties from MR-measured displacement fields of tissue, actuated at frequencies in the range of 40-1200Hz [ref]. Usually, the methods used to generate these images are derived from Navier's equation (Eq 1). The rich 3D displacement datasets provided by MR give rise to two distinct classes of inversion techniques: (1): Direct inversion (DI), where spatially homogeneous properties are assumed and the data is filtered to allow a simplified version of Navier's equation to be used. Numerical estimation of the spatial derivatives of $\vec{u}$ then allows a direct solution for the complex shear modulus[1]. (2): Non-Linear inversion (NLI), which estimates a distribution of mechanical properties by minimizing the difference between the MR measured displacements and those generated using a computational model utilizing the full non-homogeneous Navier's equation[2,3]. Viscoelastic material properties are described by a complex valued shear modulus, where the real part, μ_r, is the storage modulus, and describes the elastic stiffness, and the imaginary part, μ_i, is the loss modulus which describes the attenuation of mechanical energy. The damping ratio is then given by $\xi = \frac{\mu_i}{2\mu_r}$. Typically, estimates of μ_r are more accurate than μ_i because of differences in the sensitivity derivatives, $\frac{\partial \vec{u}}{\partial \mu_r} \gg \frac{\partial \vec{u}}{\partial \mu_i}$, which implies small fluctuations in $\vec{u}$ due to noise will have the greatest influence on μ_i. A method to correct this imbalance in sensitivity without negatively affecting the already useful μ_r reconstructions may increase the diagnostic value of attenuation parameter images.

DATA AND METHODS Traditionally, NLI properties have been supported on the same finite element mesh as the computed displacements. In this work, separate FE meshes of independent resolution are implemented for the displacements and each material property. Reconstruction of parameters with low sensitivity can be assisted by using a coarser discretization, trading off resolution against stability and quantitative accuracy. DI techniques can achieve a similar effect by using a larger window for estimation of the derivatives, however this cannot be performed independently for different parameters, and the window size for most numerical differentiation operators is limited to regions adequately described by a polynomial.

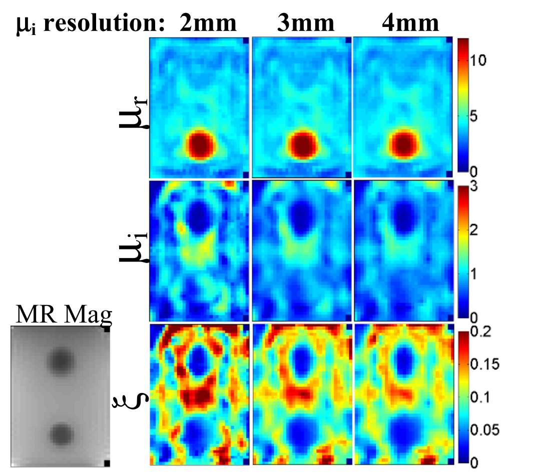

Figure 1: Multi-mesh viscoelastic reconstruction of a 2 inclusion phantom. Background = soft tofu, upper inclusion = soft gelatin, lower inclusion = hard gelatin. Gelatin is expected to have lower attenuation levels than tofu. The resolution for μ_r reconstruction was held constant at 2mm, and each column shows a different μ_i resolution.

RESULTS Figs 1 and 2 show three NLI MRE reconstructions of the same dataset, for a tofu-gel phantom and *in-vivo* brain. For each reconstruction, the μ_r resolution is held constant at 2mm, and the μ_i resolution set to either 2, 3 or 4mm. Very little change in the μ_r images is evident, no loss of resolution occurs due to changes in μ_i resolution. However, as the discretization of μ_i becomes coarser, some of the spatial variation due to noise and MR artifacts in the attenuation images (μ_i and ξ) is removed. The standard deviation of μ_i in the homogeneous background, upper and lower inclusion of the tofu-gel phantom was reduced by 39, 33 and 51% respectively in the 4mm resolution images.

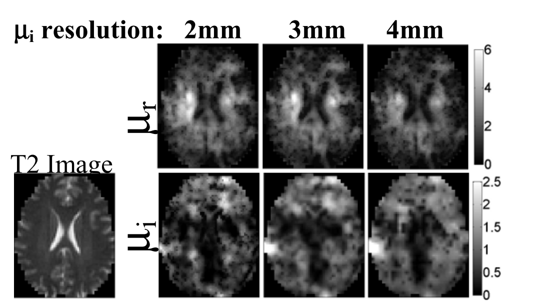

Figure 2: 50 Hz Viscoelastic reconstructions of *in-vivo* brain tissue. μ_r resolution was held constant and μ_i resolution was varied as in Fig. 1.

CONCLUSIONS Choosing each parameter resolution to suit the parameter sensitivity, data SNR and expected level of spatial variation allows low sensitivity parameters to be estimated more accurately. There is no lower limit on the property resolution, which may prove to be valuable in very low SNR MRE applications such as lung, where a very coarse grid could be used for all reconstructed parameters.

REFERENCES

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