Robust Field Map Estimation using VARPRO and Multi-labeling Continuous Max-Flow

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Introduction: The quantification of myocardial and pericardial fat is gaining clinical interest due to its association with coronary artery disease [1]. While it has been demonstrated that multi-echo Dixon methods can detect small volumes of visceral fat, field map estimation presents an obstacle for robust water/fat separation using Dixon-based techniques. The IDEAL (Iterative Decomposition of water and fat with Echo Asymmetry and Least squares estimation) method [2] provides an iterative process to simultaneously estimate the field map, water and fat components. However, this might demonstrate water/fat "swaps" in the case of abrupt changes in the field (a "swap" is defined as assigning the main signal in a water-dominant voxel as fat, or vice-versa). Several techniques have been proposed to address this problem [3-6]; however, they are either computationally expensive or less reliable in the presence of rapid changes of field offsets, such as those commonly seen near the heart. In this work, we demonstrate a novel fast field map estimation technique for which an efficient and reliable convex-relaxation algorithm called continuous max-flow (CMF) [7-8] is applied. The CMF approach addresses the proposed non-convex problem, and can be easily accelerated using modern parallel computation platforms. Our approach was tested in cardiac images obtained at 3 Tesla, where challenging cases of field inhomogeneities are more frequently encountered.

Methods: In Dixon methods, the signal $S(.)$ from each voxel is represented by $S(t_n) = \left(\rho_w + \rho_f . e^{i2\pi f_w t_n}\right). e^{i2\pi \varphi t_n}$, where t_n is the echo-time (TE) shift $(n = 1, \dots, N)$; ρ_w and ρ_f are the water and fat components, respectively; f_w (Hz) is the fat-water relative frequency shift; φ (Hz) is the local frequency offset. Using the VARPRO formulation [4], the problem is reduced to a 1-D non-convex optimization problem:

$$\min \Gamma(\varphi) := \min \left\| \left[I - \Psi(\varphi). \Psi^{\dagger}(\varphi) \right]. S \right\|_2^2 \qquad \text{where} \qquad \Psi(\varphi) = \begin{bmatrix} e^{i2\pi \varphi t_1} & e^{i2\pi (\varphi + f_w) t_1} \\ \vdots & \vdots \\ e^{i2\pi \varphi t_N} & e^{i2\pi (\varphi + f_w) t_N} \end{bmatrix},$$

I is the identity matrix, and † denotes the pseudo-inverse. By imposing a spatial smoothness on the field map, the unknown value of the field offset φ can be estimated by minimizing the following cost function: $\hat{\varphi} = \arg\min \sum_{m=1}^{M} \Gamma(\varphi) + \mu . |\nabla^3 \Gamma(\varphi)|$, where $\hat{\varphi}$ is the estimated local field offset, M is the number of voxels, and μ is a trade-off parameter to control the spatial smoothness. When acquiring images at equally spaced TEs, $\Gamma(\varphi)$ is periodic with a period of $1/\Delta TE$. Consequently, the limiting boundaries for $\hat{\varphi}$ are set to $[\pm 1/(2.\Delta TE)]$. The first stage of our approach is limiting the estimated $\hat{\varphi}$ of each voxel to a small interval (typically, 5% of the whole interval), which is obtained by assigning each voxel to a specific label using the CMF algorithm. This step locates, for each voxel, a certain range of frequency offsets where the global minimum resides. The second stage employs the labeled voxels as initial values for the IDEAL iterative process, and employs a gradient-descent based method to achieve the exact field offset value. Our two-stage approach is characterized by significantly less processing time and a robust separation in case of abrupt variations in field inhomogeneities as in cardiac images.

Results: Six cardiac datasets from healthy volunteers were acquired and compared with the region-growing technique [3]. Cardiac images were acquired with fast multi-echo GRE sequence using 32-channel torso-coil on a 3.0T (Discovery MR 750, GE Healthcare, Waukesha, WI). Data were acquired with different numbers of echo times (4, 6 and 8 echoes), to examine the reliability of the technique. A representative example is shown in the figures below, where the field map from our proposed method is compared to [3], with the corresponding water/fat separations. The data shown were acquired at 4 interleaved echoes, separated by 1.156 msec and first echo at 2.356 msec. Water/fat swaps have been clearly avoided by our approach. Moreover, the processing time is significantly reduced (~2.5 min/image vs. ~7 min/image using [3]).

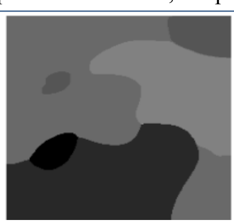

Figure 1: Output of the CMF stage, where the field offset of each pixel is assigned (labeled) to a small range of frequency offsets. This field map is used as the initial estimate of the field map in the IDEAL iterative process.

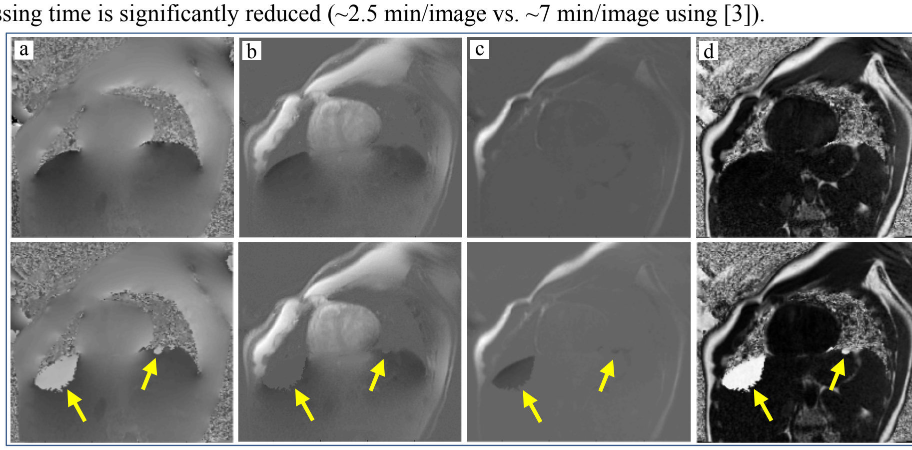

Figure 2:
Lower row shows the results using the region growing technique [3]; upper row shows the results using our proposed approach. Yellow arrows indicate the locations of water/fat swaps.
(a) Field map,
(b) water, (c) fat,
(d) fat fraction
(Fat / (Water + Fat).

Conclusion: In this work we demonstrated the feasibility of a novel technique that allows rapid and robust field map estimation. We applied a two-stage approach, where the CMF (first stage) provides a good initial estimate for the IDEAL process (second stage). The main advantages of our technique are: 1) less processing time; 2) the global optimization based algorithm helps avoid field map errors that cause water/fat swaps in the presence of sudden change in B_0, particularly in cardiac images.

References: [1] Alexopoulos, *Atherosclerosis*, 2010, 210:150. [2] Reeder, *MRM*, 2004, 51:35. [3] Yu, *MRM*, 2005, 54:1032, [4] Hernando, *MRM*, 2008, 59:571. [5] Hernando, *MRM*, 2010, 63:79. [6] Lu, *MRM,* 2008, 60:236. [7] Yuan, *ECCV*, 2010, 6316:379. [8] Bae, 2010, *UCLA Tech. Report CAM* 10-62.