Efficient concomitant field artifacts reduction using a hybrid space-frequency domain formulism

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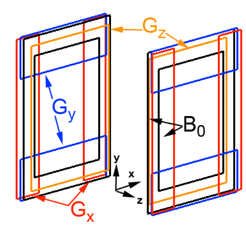
INTRODUCTION Compared to high-field MRI, ultra-low-field (ULF) MRI with B_0 in the μT range has the advantages of 1) compatibility with metal objects, 2) system of a lighter weight and a lower cost, 3) higher T_1 contrast, and 4) taking MRI and magnetoencephalography (MEG) simultaneously [1]. One difference of ULF MRI reconstruction from high-field MRI is the non-negligible concomitant field artifacts [2]. Considering these artifacts, the spatially encoded ULF MRI signal can be related to the spatial distribution of the magnetization using a linear equation [3], whose encoding matrix is too large to be practically inverted for image reconstruction. Alternatively, local magnetization phase in the encoding matrix can be first calibrated and subsequently reconstructed using Fourier transform [4]. However, this method cannot accurately describe the magnetization dynamics when the direction of the magnetic field making phase encoding is either (*i*) not in parallel with the magnetic field during read-out or (*ii*) not perpendicular to the initial magnetization.

Here we propose a two-stage space-frequency (*x-f*) hybrid formalism to accurately describe the dynamics of spatially encoded magnetization allowing arbitrary directions of the initial magnetization and phase/frequency encoding magnetic fields. This is particularly useful in ULF MRI where concomitant field artifacts can no longer be neglected. Compared to the full time-domain solver, the complexity of a 2D image reconstruction of *n* image pixels is reduced from $O(n^3)$ to $O(n^2)$ using fast Fourier transformed data and a linear equation solver. We present this method together with numerical simulations to demonstrate how concomitant field artifacts in ULF MRI can be corrected.

METHODS To generate spatial encoded ULF MRI signal using a gradient $\mathbf{g}$, $\nabla b_z(\mathbf{r}) = \mathbf{g} = [g_x\ g_y\ g_z]^T$, the total magnetic field $\mathbf{b}(\mathbf{r})$ consists of an ideal linear part $\mathbf{b}_{ideal}(\mathbf{r}) = (B_0 + \mathbf{g}^T\mathbf{r})\,\mathbf{e}_z$, and a concomitant field part $\mathbf{b}_{con}(\mathbf{r}) = (-g_z x/2 + g_x z)\mathbf{e}_x + (-g_z y/2 + g_y z)\mathbf{e}_y$, where B_0 is the strength of the measurement field, and $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are spatial unit vectors. We define $\varepsilon = g_{max}\mathrm{FOV}/B_0$ to quantify the effect of the concomitant field, where g_{max} is the maximal amplitude among $[g_x\ g_y\ g_z]$ over the whole experiment. Typically, ULF MRI has $0.1 < \varepsilon < 1$ and thus the concomitant fields $\mathbf{b}_{con}(\mathbf{r})$ cannot be ignored. To accurately describe the dynamics of magnetization precession in the read-out of an *n*-step phase-encoded ULF MRI experiment ($1 \le n \le N$), we first define a unit vector $\mathbf{e}_m(\mathbf{r}, n)$ as the direction of the local magnetization $\mathbf{m}(\mathbf{r},n) = \rho(\mathbf{r})\mathbf{e}_m(\mathbf{r}, n)$ at the beginning of the read-out, where $\rho(\mathbf{r})$ denotes the spin density at location $\mathbf{r}$. The unit vector $\mathbf{e}_b(\mathbf{r})$ denotes the direction of $\mathbf{b}(\mathbf{r})$, and $\mathbf{e}_s$ is the unit vector of the local sensitivity $\mathbf{s}(\mathbf{r})$ of a pick-up coil. The detectable non-DC signal at time *t* becomes the spatial integration of $\{\rho(\mathbf{r})\, m_\perp(\mathbf{r}, n)\cos(\gamma|\mathbf{b}(\mathbf{r})|t + \phi(\mathbf{r},n))s_\perp(\mathbf{r})\}$ over the whole FOV, where $m_\perp(\mathbf{r}, n)$ and $s_\perp(\mathbf{r})$ are the magnitude of $\mathbf{e}_m(\mathbf{r}, n)$ and $\mathbf{s}(\mathbf{r})$ perpendicular to $\mathbf{e}_b(\mathbf{r})$. $\phi(\mathbf{r},n)$ is the angle between the projected component of $\mathbf{e}_m(\mathbf{r}, n)$ and $\mathbf{s}(\mathbf{r})$ over a plane with its normal vector $\mathbf{e}_b(\mathbf{r})$.

Here, we propose that it is actually possible to separate the image reconstruction into two independent stages: first we use Fourier transform to estimate the distribution of magnetization precession spectra distributed over an iso-frequency curve in the real space for each frequency-encoded read-out. In this case, a linear equation describing a generalized phase encoding allows quantitative description of (*i*) the detectable component of the phase-encoded magnetization $m_\perp$ as a function of $\mathbf{r}$ and (ii) a nonlinear relationship between the phases and the spatial coordinates along an iso-frequency curve. This is different from traditional MRI using idealized linear gradients, where $m_\perp$ is constant such that there is a linear relationship between magnetization phases and spatial coordinates along an iso-frequency straight line. Specifically, the Fourier transform of the *signal*(*t,n*) is

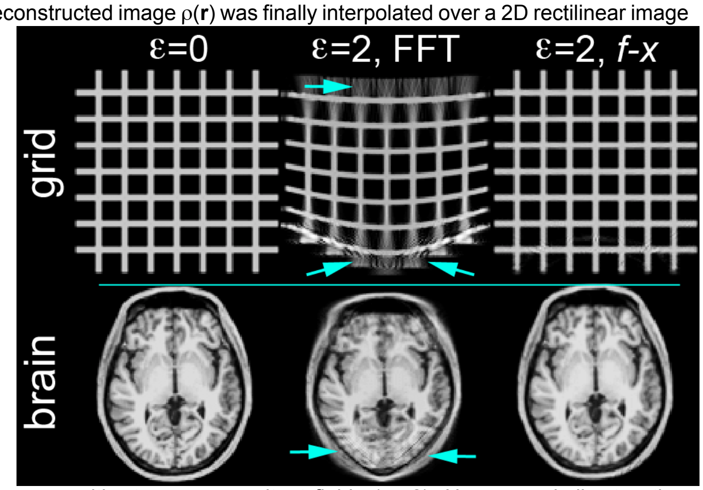
$$signal(\omega,n) = \int_{\gamma|b(\mathbf{r})|=\omega} \rho(\mathbf{r}) m_\perp(\mathbf{r},n) \exp(-\phi(\mathbf{r},n)) s_\perp(\mathbf{r}) J(\mathbf{r})\, ds$$



Here we describe the signal as a function of frequency ω generated from the spatial integration of all magnetizations with a precession frequency $\gamma|\mathbf{b}(\mathbf{r})| = \omega$. $J(\mathbf{r}) = 1/|\nabla(\gamma|\mathbf{b}(\mathbf{r})|)|$ is the Jacobian corresponding to changes between Cartesian coordinate $\mathbf{r}$ and frequency coordinate ω.

At each frequency ω_0, *N* measurements $signal(\omega|\omega=\omega_0, n)$ were used to solve for the spatial distribution of $\rho(\mathbf{r}|\gamma|\mathbf{b}(\mathbf{r})|=\omega_0)$. Along a curve in the image domain whose Larmor frequency is ω_0, we solved for $\rho(\mathbf{r}|\gamma|\mathbf{b}(\mathbf{r})|=\omega_0)$ over 2*N* locations in order to capture potential spatial phase variations faster than what is expected from the ideal linear gradients due to concomitant fields. Practically we imposed a prior constraint to minimize the L2 norm of $\mathbf{D}\rho(\mathbf{r})$, where $\mathbf{D}$ takes the difference between two neighboring data points. The same procedure was repeated for different ω_0 to sweep over all frequencies of magnetization precession within the desired FOV in the read-out direction. The reconstructed image $\rho(\mathbf{r})$ was finally interpolated over a 2D rectilinear image grid.

Our experimental setup (top figure) consists of coils for $B_0 = 50$ μT (black), three gradient coils (red, orange, and blue) and a polarizing coil (not shown). Magnetic fields were calculated using the Biot–Savart's law given the coil geometry. To focus on the concomitant field artifacts, we set $s_\perp(\mathbf{r}) = 1$ in this study. Concomitant fields were simulated in the cases of $\varepsilon = 0$ ($B_0 = \infty$), and $\varepsilon = 2$. The *signal*(*t,n*) over 128 phase encoding steps was calculated by numerical integration using 4096x4096 spatial grid points. All calculations were implemented on a PC in Matlab (Mathworks, Natick, MA).



RESULTS The computational time for the simulation was 2 s. The figure at right shows the reconstruction of a grid and brain phantoms in the cases of no concomitant field ($\varepsilon = 0$) and a strong concomitant field ($\varepsilon = 2$). In contrast to reconstructions using fast Fourier transform (FFT), the *f-x* hybrid method reduces the concomitant field blurring, image compression (horizontal cyan line), and distortion (cyan arrows) artifacts at the top and bottom of the FOV.

DISCUSSION The presented *x-f* hybrid method reconstructs images efficiently even with strong concomitant fields ($\varepsilon = 2$). However, similar to other methods, if concomitant fields cause non-uniform distribution of $\nabla|\mathbf{b}(\mathbf{r})|$, the loss in the spatial resolution cannot be corrected. The regularization parameter used in solving the linear equation should be tuned based on the condition of the encoding matrix in order to obtain the best results. While we demonstrated this *x-f* hybrid method in ULF MRI reconstructions, the method can be applied to nonlinear encoded high-field MRI, too.

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